Aufgabe 1b)

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle = e^{-|\alpha|^2/2} e^{\alpha a^+} |0\rangle$$

$$P(\alpha, \alpha^*) = \frac{e^{|\alpha|^2}}{\pi^2} \int d^2\beta \, \langle -\beta | \rho | \beta \rangle \, e^{|\beta|^2} e^{-\beta\alpha^* + \beta^*\alpha} d^2\beta$$

$\langle -\beta$ $\rho = |n\rangle\langle n|$

$$\langle -\beta | \rho | \beta \rangle = e^{-|\beta|^2} \frac{(-1)^n |\beta|^{2n}}{n!}$$

$$P(\alpha, \alpha^*) = \frac{e^{|\alpha|^2}}{\pi^2 n!} \frac{\partial^{2n}}{\partial\alpha^n \partial\alpha^{*n}} \int d^2\beta \, e^{-\beta\alpha^* + \beta^*\alpha}$$

$$= \frac{e^{|\alpha|^2}}{n!} \frac{\partial^{2n}}{\partial\alpha^n \partial\alpha^{*n}} \delta^{(2)}(\alpha)$$

1a) $$Q(\alpha, \alpha^*) = \frac{1}{\pi} \langle \alpha | \rho | \alpha \rangle = \frac{1}{\pi} |\langle n | \alpha \rangle|^2$$

$$= \frac{1}{\pi} |\langle n | \alpha \rangle|^2 = \frac{1}{\pi n!} |\langle 0 | a^n | \alpha \rangle|^2$$

$$= \frac{|\alpha|^{2n}}{\pi n!} e^{-|\alpha|^2}$$

Aufgabe 1b)

$$W(\alpha,\alpha^*) = \frac{2}{\pi}\int d^2\lambda \; e^{-2|\lambda-\alpha|^2} \; P(\lambda,\lambda^*)$$

$$= \frac{2}{\pi}\frac{1}{n!} e^{-2|\alpha|^2} \frac{\partial^{2n}}{\partial\lambda^n \, \partial\lambda^{*n}} e^{-2|\lambda|^2} e^{2\lambda\alpha^*} e^{2\lambda^*\alpha} \Big|_{\lambda=\lambda^*=}$$

$$\frac{\partial^{2n}}{\partial\lambda^n \, \partial\lambda^{*n}} e^{-A|\lambda|^2} e^{B\lambda} e^{C\lambda^*} = e^{B\lambda} e^{C\lambda^*} \left(B + \frac{\partial}{\partial\lambda}\right)^n \left(C + \frac{\partial}{\partial\lambda^*}\right)^n e^{-A|\lambda|^2}$$

$$= e^{B\lambda} e^{C\lambda} \left(B + \frac{\partial}{\partial\lambda}\right)^n (C - A\lambda)^n e^{-A|\lambda|^2}$$

$$\left(B + \frac{\partial}{\partial\lambda}\right)^n (C - \lambda A)^n$$

$$= \left(B + \frac{\partial}{\partial\lambda}\right)^{n-l} \sum_{k=0}^{l} \frac{l!}{k(l-k)!} \frac{n!}{(n-k)!} A^k (C - \lambda A)^{n-k}$$

$$\times \left(B + \frac{\partial}{\partial\lambda}\right)^{n-k}$$

$$A = 2, \; B^* = C = 2\alpha$$

$$W(\alpha,\alpha^*) = \frac{2}{\pi}\frac{1}{n!} e^{-2|\alpha|^2} \sum_{k=0}^{n} (-1)^{n-k} \frac{n!}{(n-k)!k!} \frac{n!}{k!}$$

$$\times |2\alpha|^{2k}$$

Aufgabe 2:

$$\frac{\partial}{\partial\alpha} |\alpha\rangle\langle\alpha| = \frac{\partial}{\partial\alpha}\left( e^{-\overset{\alpha\alpha^*}{|\alpha|^2}} e^{\alpha a^+} |0\rangle\langle 0| e^{\alpha^* a}\right)$$

$$= (a^+ - \alpha^*)(|\alpha\rangle\langle\alpha|$$

$$\frac{\partial}{\partial\alpha^*} |\alpha\rangle\langle\alpha| = \frac{\partial}{\partial\alpha^*}\left( e^{-|\alpha|^2} e^{\alpha a^+} |0\rangle\langle 0| e^{\alpha^* a}\right)$$

$$= |\alpha\rangle\langle\alpha|(a - \alpha)$$

Aufgabe 3

$$\dot{\rho} = -i\omega_0 (a^+ a \rho - \rho a^+ a)$$

$$\int d^2\alpha \frac{\partial}{\partial t} P(\alpha, t) |\alpha\rangle\langle\alpha| =$$

$$\int d^2\alpha \; P(\alpha, t) \left[ -i\omega_0 \left( a^+ a |\alpha\rangle\langle\alpha| - |\alpha\rangle\langle\alpha| a^+ a \right) \right]$$

$$a^+ a |\alpha\rangle\langle\alpha| = \alpha a^+ |\alpha\rangle\langle\alpha|$$

$$= \alpha \left( \frac{\partial}{\partial\alpha} + \alpha^* \right) |\alpha\rangle\langle\alpha|$$

$$|\alpha\rangle\langle\alpha| a^+ a = \alpha^* |\alpha\rangle\langle\alpha| a \Rightarrow \alpha$$

$$= \alpha^* \left( \frac{\partial}{\partial\alpha^*} + \alpha \right) |\alpha\rangle\langle\alpha|$$

Fortsetzung Aufg 3:

$$\int d^2\alpha \, |\alpha\rangle\langle\alpha| \frac{\partial}{\partial t} P(\alpha,t)$$

$$\cancel{= \int d^2\alpha \, P(\alpha,t) \left[ -\left( \frac{\gamma}{2} + i\omega_0 \right) \alpha \, \partial \right.}$$

$$= \int d^2\alpha \, P(\alpha,t) \, i\omega_0 \left[ -\alpha \frac{\partial}{\partial\alpha} + \alpha^* \frac{\partial}{\partial\alpha^*} \right] |\alpha\rangle\langle\alpha|$$

$$\int f g' \, dx = [f g] - \int g f' \, dx$$

$$\int d^2\alpha \, |\alpha\rangle\langle\alpha| \frac{\partial}{\partial t} P(\alpha,t)$$

$$= \int d^2\alpha \, \cancel{P(\alpha,t)} \, |\alpha\rangle\langle\alpha| \, i\omega_0 \left[ \alpha \frac{\partial}{\partial\alpha} - \alpha^* \frac{\partial}{\partial\alpha^*} \right] P(\alpha,t)$$

$$\frac{\partial}{\partial t} P(\alpha,t) = i\omega_0 \left[ \alpha \frac{\partial}{\partial\alpha} - \alpha^* \frac{\partial}{\partial\alpha^*} \right] P(\alpha,t)$$