Tafelanschrieb:

$$\hat{H} = 4E_C(-i\frac{d}{d\varphi} - n_g)^2 - E_J\cos\hat{\varphi}$$

(i) eliminiere n_g durch Eichtransformation

(ii) entwickle $\cos\hat{\varphi} \approx 1 - \frac{\hat{\varphi}^2}{2} + \frac{\hat{\varphi}^4}{24} - +...$

$$\rightarrow \hat{H} = 4E_C \underbrace{(-i\frac{d}{d\varphi})^2}_{\hat{p}} + E_J \underbrace{\frac{\hat{\varphi}^2}{2}}_{\hat{\varphi} \doteq \hat{x}} - E_J - E_J\frac{\hat{\varphi}^4}{24}$$

vgl. harm. Oszillator mit Störung:

$$\rightarrow \hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{x}^2}{2} - m\omega^2 - m\omega^2\frac{\hat{\varphi}^4}{24} \qquad \text{mit}: \quad m = \frac{1}{8E_C} \quad \omega = \sqrt{8E_JE_C}$$

mit $\hat{\varphi} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{b} + \hat{b}^\dagger) = \left(\frac{2E_C\hbar^2}{E_J}\right)^{1/4}(\hat{b} + \hat{b}^\dagger) \qquad \hat{p} = \frac{1}{i}\sqrt{\frac{m\omega\hbar}{2}}(\hat{b} - \hat{b}^\dagger) = \frac{1}{i}\left(\frac{E_J\hbar^2}{32E_C}\right)^{1/4}(\hat{b} - \hat{b}^\dagger)$

$$\rightarrow \hat{H} = \hbar\omega(\hat{b}^\dagger\hat{b} + \frac{1}{2}) - E_J - \frac{E_C}{12}(\hat{b} + \hat{b}^\dagger)^4$$

Korrektur zu Eigenenergien des harm. Oszillator:

$$E_j^{(1)} = -\frac{E_C}{12}\langle j| (\hat{b} + \hat{b}^\dagger)^4 |j\rangle = -\frac{E_C}{12}(6j^2 + 6j + 3)$$

$$\rightarrow E_m = \hbar\sqrt{8E_JE_C}(m + \frac{1}{2}) - E_J - \frac{E_C}{12}(6m^2 + 6m + 3)$$