Karlsruher Institut für Technologie (KIT) SS2012
Institut für theoretische Festkörperphysik
Prof. Dr. G. Schön, Dr. R. Frank **15.06.2012**
http://www.tfp.kit.edu/studium-lehre.php schoen@kit.edu,rfrank@tfp.uni-karlsruhe.de

**Tutorial:**

☐ **Group 1,**

☐ **Group 2,**

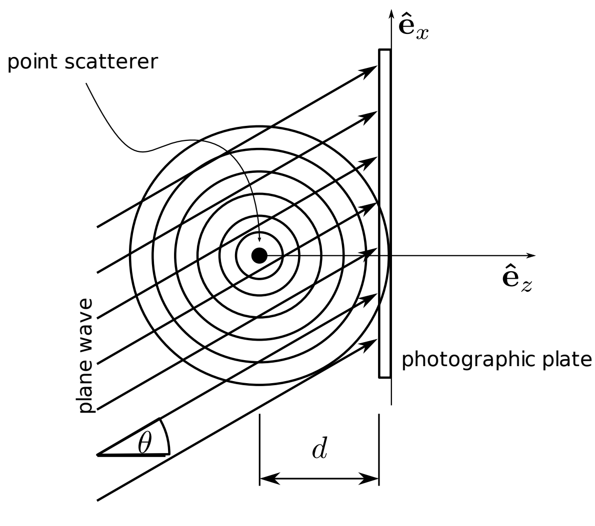
☐ **Group 3.**

**Name:**________________________________________

# Problem set 6 for the course "Theoretical Optics" Sample Solutions

## 12 Hologram Of A Point Scatterer

A point scatterer resides on the z-axis in distance $d$ to a photographic plate at $z = 0$. A plane reference wave $\Psi_{\text{ref}}(\mathbf{r}) = A_{\text{ref}} e^{-ik_0 x \sin\theta}$ interferes on the screen with the object wave $\Psi_{\text{obj}}(x, y) = \frac{A_{\text{obj}}}{\sqrt{x^2+y^2+d^2}} e^{-ik_0\sqrt{x^2+y^2+d^2}}$, where $A_{\text{ref}}, A_{\text{obj}} \in \mathbb{R}$. We consider a large distance $d$ between object and screen.




**a)** We want to calculate the interference effects on the screen in the following. Therefore, find an approximate expression of the object wave $\Psi_{\text{obj}}$ for large distance $d$ between object and screen. Explain, what exactly this property means mathematically. Explain, why this allows You to approximate the square roots by Taylor polynomials then and why it suffices to expand the amplitude up to zeroth order in $x$ and $y$ (means that the approximation is independent of $x$ and $y$), while for the argument of the exponential the next non-vanishing order of the variables $x$ and $y$ has to be included. [**2** Point(s)]

**b)** The photographic screen will show the interference pattern of the reference and object wave after developing, thus acting as an aperture with amplitude transmission coefficient $T(x, y) = |\Psi_{\text{obj}} + \Psi_{\text{ref}}|^2$. Show that

$$T(x, y) = A_{\text{ref}}^2 + \frac{A_{\text{obj}}^2}{d^2} + 2\frac{A_{\text{ref}}A_{\text{obj}}}{d} \cos\left(k_0 d - \frac{k_0 d \sin^2\theta}{2} + \frac{k_0}{2d}\left((x - d\sin\theta)^2 + y^2\right)\right). \quad (1)$$

[**3** Point(s)]

**c)** To reconstruct the image of the point scatterer, we illuminate the developed screen with an incoming plane wave $\Psi_{\text{in}} = \Psi_{\text{ref}}$ of the same frequency as the reference wave. Directly behind the screen, the transmitted field distribution is then given by $\Psi_{\text{trans}} = \Psi_{\text{in}} T(x, y)$. Find the field $\Psi_{\text{trans}}(x, y)$ on the screen for general $\theta$ in terms of propagating waves (express sin and cos by complex exponential functions). Which of these partial waves reconstructs the image of the point scatterer? What is the physical meaning of the remaining partial waves?[**3** Point(s)]

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**a)** The screen coordinates are given by $x$ and $y$, where the screen is supposed to have finite extents along these coordinates, e.g., $h$ and $w$ (not given in the problem text). A large distance $d$ then means, that $d$ is always much larger than any values for $x$ and $y$ that we use implicitly to evaluate any of the wave expressions, i.e. $\boxed{d \gg x, y}$ [**0.5** Point(s)].

Now we write the square root as

$$\sqrt{x^2 + y^2 + d^2} = d\sqrt{1 + \underbrace{\frac{x^2 + y^2}{d^2}}_{\ll 1}}, \quad (2)$$

which is basically the function $f(t) = \sqrt{1 + t^2}$ evaluated for small positive $t \ll 1$ only. Hence, the Taylor expansion around the point 0 is a good approximation to the values of this function $f$, which up to first order in the argument gives

$$\sqrt{1 + t} \approx 1 + \frac{1}{2}t + O(t^2). \qquad \text{(Taylor around } t_0 = 0 \text{ for } t \ll 1) \quad (3)$$

Thus, at $t = \frac{x^2+y^2}{d^2}$, we obtain the expansion

$$\sqrt{x^2 + y^2 + d^2} \approx d + \frac{x^2 + y^2}{2d} + O\left(\frac{1}{d^3}\right). \quad (4)$$

In the expression

$$\Psi_{\text{obj}}(x, y) = \frac{A_{\text{obj}}}{\sqrt{x^2 + y^2 + d^2}} \mathrm{e}^{-\mathrm{i}k_0\sqrt{x^2+y^2+d^2}}, \quad (5)$$

we need to expand the square roots now. As discussed in the lecture, it is usually sufficient to only keep the first term in the amplitude while the phase is much more sensitive, so there we should also keep the second term as well (Fresnel approximation). This is because the amplitude variations are not as important as the phase variations for obtaining an interference pattern (if we approximated the phase to be independent of $x$ and $y$, we could not study the interference effects, because we would have 'approximated it to death'). Thus, we write

$$\Psi_{\text{obj}}(x, y) \approx \frac{A_{\text{obj}}}{d} \mathrm{e}^{-\mathrm{i}k_0 d} \mathrm{e}^{-\mathrm{i}\frac{k_0}{2d}(x^2+y^2)}, \quad (6)$$

which is the desired asymptotic behavior of the object wave on the far away screen that allows to study the most prominent interference contributions.

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**b)** Starting from the original expression, we find

$$T(x,y) = |\Psi_{\text{obj}} + \Psi_{\text{ref}}|^2 \tag{7}$$
$$= |\Psi_{\text{obj}}|^2 + |\Psi_{\text{ref}}|^2 + \underbrace{\Psi^*_{\text{ref}}\Psi_{\text{obj}} + \Psi_{\text{ref}}\Psi^*_{\text{obj}}}_{2\text{Re}\left(\Psi_{\text{ref}}\Psi^*_{\text{obj}}\right)} \tag{8}$$

So, we need to evaluate

$$2\text{Re}\left(\Psi_{\text{ref}}\Psi^*_{\text{obj}}\right) = 2\text{Re}\left(\frac{A_{\text{obj}}A_{\text{ref}}}{d}\text{e}^{-\text{i}k_0\left(x\sin\theta - d - \frac{x^2+y^2}{2d}\right)}\right) \qquad \text{(entered given wave expressions)} \tag{9}$$

$$= 2\frac{A_{\text{obj}}A_{\text{ref}}}{d}\cos\Big(k_0\big(x\sin\theta - d - \frac{x^2+y^2}{2d}\big)\Big) \qquad \text{(took real part)} \tag{10}$$
$$= 2\frac{A_{\text{obj}}A_{\text{ref}}}{d}\cos\Big(k_0\big(d + \frac{x^2+y^2}{2d} - x\sin\theta\big)\Big). \qquad (\cos(-\theta) = \cos(\theta)) \tag{11}$$

Adding a 0 allows to recast the expression as

$$= 2\frac{A_{\text{obj}}A_{\text{ref}}}{d}\cos\left(k_0 d + k_0\frac{x^2+y^2}{2d} - k_0 x\sin\theta + \underbrace{\frac{k_0 d}{2}\sin^2\theta - \frac{k_0 d}{2}\sin^2\theta}_{=0}\right) \tag{12}$$

rearranging terms gives

$$= 2\frac{A_{\text{obj}}A_{\text{ref}}}{d}\cos\left(k_0 d - \frac{k_0 d}{2}\sin^2\theta + \frac{k_0}{2d}\left(y^2 + \underbrace{x^2 - 2xd\sin\theta - d^2\sin^2\theta}_{=(x-d\sin\theta)^2}\right)\right) \tag{13}$$
$$= 2\frac{A_{\text{obj}}A_{\text{ref}}}{d}\cos\left(k_0 d - \frac{k_0 d}{2}\sin^2\theta + \frac{k_0}{2d}\left((x-d\sin\theta)^2 + y^2\right)\right). \tag{14}$$

We therefore find that the interference pattern is centered around $x - d\sin\theta$, as sugested by looking at the sketch.

Finally, the full expression for $T$ reads

$$T(x,y) = |\Psi_{\text{obj}} + \Psi_{\text{ref}}|^2 \tag{15}$$
$$= |\Psi_{\text{obj}}|^2 + |\Psi_{\text{ref}}|^2 + 2\text{Re}\left(\Psi_{\text{ref}}\Psi^*_{\text{obj}}\right) \tag{16}$$
$$= \left(\frac{A_{\text{obj}}}{d}\right)^2 + A^2_{\text{ref}} + 2\frac{A_{\text{obj}}A_{\text{ref}}}{d}\cos\left(k_0 d - \frac{k_0 d}{2}\sin^2\theta + \frac{k_0}{2d}\left((x-d\sin\theta)^2 + y^2\right)\right). \tag{17}$$

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**c)** So, we write the transmitted field (directly behind the screen) as:

$$\begin{aligned}
\Psi_{\text{trans}} &= \Psi_{\text{ref}} T(x,y) && (18)\\
&= \Psi_{\text{ref}} |\Psi_{\text{obj}} + \Psi_{\text{ref}}|^2 && (19)\\
&= \Psi_{\text{ref}} \left( |\Psi_{\text{obj}}|^2 + |\Psi_{\text{ref}}|^2 + \Psi_{\text{ref}}\Psi^*_{\text{obj}} + \Psi^*_{\text{ref}}\Psi_{\text{obj}} \right) && (20)\\
&= \underbrace{\left( \left(|\Psi_{\text{obj}}|^2 + |\Psi_{\text{ref}}|^2\right) \Psi_{\text{ref}} \right)}_{\text{term 1}} + \underbrace{\Psi^2_{\text{ref}}\Psi^*_{\text{obj}}}_{\text{term 2}} + \underbrace{|\Psi_{\text{ref}}|^2\Psi_{\text{obj}}}_{\text{term 3}} && (21)
\end{aligned}$$

From this expression, we find already, that the first two partial wave in term 1 simply reconstruct the reference beam (with a modified amplitude). Similarly, term 3 corresponds to the object wave (outgoing spherical wave) with a modified amplitude.

To interpret the remaining term 2 we need to investigate a little bit further. Inserting the full expressions we have

$$\Psi_{\text{trans}} = \Psi_{\text{inc}} T(x,y) = A_{\text{ref}} \mathrm{e}^{-\mathrm{i}k_0 x \sin\theta} T(x,y)$$

with $T$ from above (here we use the simpler form, not the final result) as

$$\begin{aligned}
T(x,y) &= (\frac{A_{\text{obj}}}{d})^2 + A^2_{\text{ref}} + \frac{A_{\text{obj}}A_{\text{ref}}}{d} 2\cos\left( k_0 \left( d + \frac{x^2+y^2}{2d} - x\sin\theta \right) \right) && (22)\\
&= (\frac{A_{\text{obj}}}{d})^2 + A^2_{\text{ref}} + \frac{A_{\text{obj}}A_{\text{ref}}}{d} \left( \mathrm{e}^{+\mathrm{i}\left(k_0\left(d+\frac{x^2+y^2}{2d} - x\sin\theta\right)\right)} + \mathrm{e}^{-\mathrm{i}\left(k_0\left(d+\frac{x^2+y^2}{2d} - x\sin\theta\right)\right)} \right), && (23)
\end{aligned}$$

we find:

$$\begin{aligned}
\Psi_{\text{trans}} &= \left( (\frac{A_{\text{obj}}}{d})^2 + A^2_{\text{ref}} \right) A_{\text{ref}} \mathrm{e}^{-\mathrm{i}k_0 x \sin\theta} && (24)\\
&+ \frac{A^2_{\text{ref}}A_{\text{obj}}}{d} \mathrm{e}^{-\mathrm{i}k_0 d} \mathrm{e}^{-\mathrm{i}k_0 \frac{x^2+y^2}{2d}} \mathrm{e}^{\mathrm{i}k_0 x\sin\theta} \mathrm{e}^{-\mathrm{i}k_0 x \sin\theta} && (25)\\
&+ \frac{A^2_{\text{ref}}A_{\text{obj}}}{d} \mathrm{e}^{\mathrm{i}k_0 d} \mathrm{e}^{\mathrm{i}k_0 \frac{x^2+y^2}{2d}} \mathrm{e}^{-\mathrm{i}2k_0 x \sin\theta} && (26)\\
&= \left( (\frac{A_{\text{obj}}}{d})^2 + A^2_{\text{ref}} \right) A_{\text{ref}} \mathrm{e}^{-\mathrm{i}k_0 x \sin\theta} && \text{term 1} \quad (27)\\
&+ \frac{A^2_{\text{ref}}A_{\text{obj}}}{d} \mathrm{e}^{-\mathrm{i}k_0 d} \mathrm{e}^{-\mathrm{i}k_0 \frac{x^2+y^2}{2d}} && \text{term 3} \quad (28)\\
&+ \frac{A^2_{\text{ref}}A_{\text{obj}}}{d} \mathrm{e}^{\mathrm{i}k_0 d} \mathrm{e}^{\mathrm{i}k_0 \frac{(x-2d\sin\theta)^2+y^2}{2d}} \mathrm{e}^{-\mathrm{i}2k_0 d \sin^2\theta} && \text{term 2} \quad (29)
\end{aligned}$$

Summary of the various terms:

1. The image of the reference beam (plane wave pattern on the screen)
2. Incoming spherical wave from position $z = +d$ (in front of the screen) at ($x = 2d\sin\theta, y = 0$). This is a real image of the scatterer. In this image, depth information is reversed, i.e. bumps are dents and vice versa. That's why it is also called the pseudoscopic image (the name denotes the depth reversion). See Fig. **??**.
3. Outgoing spherical wave of the point scatterer. This corresponds to a virtual image of our scatterer at its original position ($x = 0, y = 0$) in distance $z = -d$ (behind the screen). This is the hologram.

Thus, (Gabor) holograms simultaneously generate both a virtual and a real image, also called twin images.

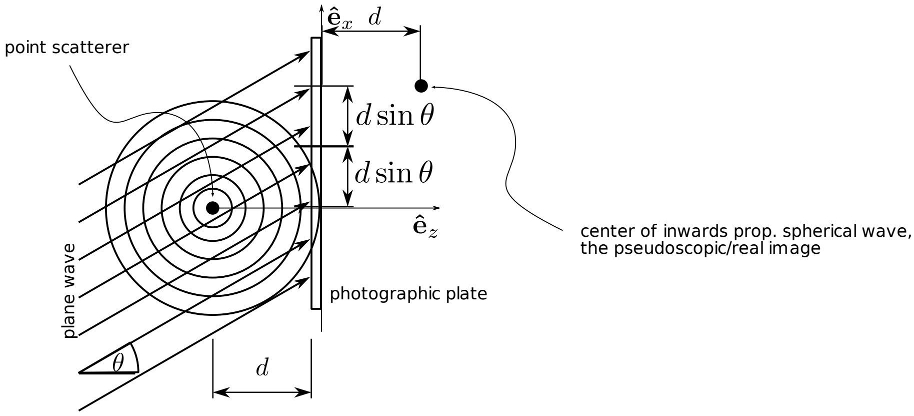


Figure 1: The hologram of a point and its pseudoscopic image.

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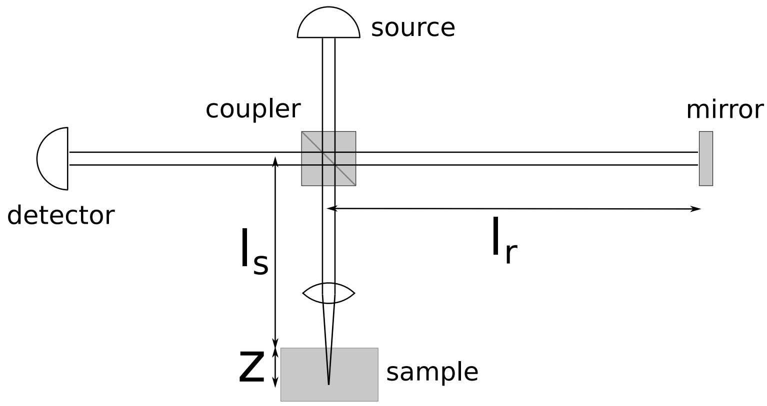
## 13 Optical Coherence Tomography

Optical coherence tomography is a scanning optical microscope with improved vertical resolution.

From a source, light with amplitude $E_0(t)$ emerges and is split into a reference ray and a measurement ray. This is focused by a conventional microscope into a sample with mean refractive index $\bar{n}$. $E_\mathrm{r}(t)$ is the amplitude of the reflected reference ray at the position of the coupler; $E_\mathrm{sc}(z;t)$ is the amplitude of the light scattered inside the sample at the position of the coupler. The light is scattered at some particle located at depth $z$ with the real–valued scattering efficiency $a_\mathrm{sc}$.




**a)** Formulate $E_\mathrm{r}(t)$ and $E_\mathrm{sc}(z;t)$ in terms of $E_0(t)$. Express the detected intensity $I_\mathrm{det} = \left\langle \left| E_\mathrm{r}(t) + E_\mathrm{sc}(t) \right|^2 \right\rangle$ in terms of the autocorrelation function $\Gamma(\tau) = \langle E_0(t) E_0^*(t-\tau) \rangle$. [**4** Point(s)]

**b)** Assume that $E_0$ has a Gaussian spectral power density with mean angular frequency $\omega_0$ and width $\Delta\omega$:

$$\left| E_0(\omega) \right|^2 = i_0 \cdot \exp\left( -\frac{(\omega - \omega_0)^2}{2\Delta\omega^2} \right) \tag{30}$$

Find $\left| \Gamma(\tau) \right|$. [**3** Point(s)]

**c)** Consider a single point scatterer at $z = z_{\text{sc}}$. Calculate $I_{\text{det}}$ as a function of $s := 2l_r - 2l_s$ using the results of **a)** and **b)**. Sketch $I_{\text{det}}(s)$ and annotate the main features. [**3** Point(s)]

**d)** Which features of the interferometer and of $I_{\text{det}}(s)$ have to be known to determine the quantities $z_{\text{sc}}$ and $a_{\text{sc}}$? How does the light source influence the vertical resolution (z-direction)? What determines the lateral resolution (x-direction)?[**2** Point(s)]

OCT is used in eye diagnostics and more general in biology and medicine as an imaging method. It allows for good $z$-resolution (depth) in the images. See Wikipedia for more details.

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**a)** $E_{\text{r}}(t)$ is the light that was reflected at the reference mirror. Hence, by conservation of energy (intensity is half) it's delayed by $\frac{2l_{\text{r}}}{c}$ with a relative amplitude of $1/\sqrt{2}$:

$$E_{\text{r}}(t) = \frac{1}{\sqrt{2}} E_0(t - \tau_{\text{r}}), \tag{31}$$

$$\tau_{\text{r}} = \frac{2l_{\text{r}}}{c}. \tag{32}$$

Analogously, the light scattered inside the sample is delayed by $\frac{2l_{\text{s}}}{c}$ accounting for the free space until it enters the sample and $\frac{2z\bar{n}}{c}$ accounting for the propagation inside the sample:

$$E_{\text{sc}}(t) = \frac{1}{\sqrt{2}} a_{\text{sc}} E_0(t - \tau_{\text{sc}}), \tag{33}$$

$$\tau_{\text{sc}} = \frac{2l_{\text{s}} + 2z\bar{n}}{c}. \tag{34}$$

The scattered amplitude is furthermore multiplied with the scattering efficiency.

The intensity at the detector is given by

$$\begin{aligned}
I_{\text{det}} &= \left\langle \left| E_{\text{r}}(t) + E_{\text{sc}}(t) \right|^2 \right\rangle & (35)\\
&= \left\langle \left( E_{\text{r}}(t) + E_{\text{sc}}(t) \right) \left( E_{\text{r}}^*(t) + E_{\text{sc}}^*(t) \right) \right\rangle & (36)\\
&= \left\langle E_{\text{r}}(t) E_{\text{r}}^*(t) + E_{\text{sc}}(t) E_{\text{sc}}^*(t) + 2\text{Re}\left\{ E_{\text{r}}(t) E_{\text{sc}}^*(t) \right\} \right\rangle & (37)\\
&= \left\langle E_{\text{r}}(t) E_{\text{r}}^*(t) \right\rangle + \left\langle E_{\text{sc}}(t) E_{\text{sc}}^*(t) \right\rangle + \left\langle 2\text{Re}\left\{ E_{\text{r}}(t) E_{\text{sc}}^*(t) \right\} \right\rangle & (38)\\
&= \frac{1}{2} \Big[ \left\langle E_0(t - \tau_{\text{r}}) E_0^*(t - \tau_{\text{r}}) \right\rangle + \left\langle a_{\text{sc}} E_0(t - \tau_{\text{sc}}) a_{\text{sc}} E_0^*(t - \tau_{\text{sc}}) \right\rangle \\
&\quad + 2\text{Re}\left\{ \left\langle E_0(t - \tau_{\text{r}}) a_{\text{sc}} E_0^*(t - \tau_{\text{sc}}) \right\rangle \right\} \Big] & (39)\\
&= \frac{1}{2} [\Gamma(0) + a_{\text{sc}}^2 \Gamma(0)] + \text{Re}\left\{ a_{\text{sc}} \Gamma(\tau_{\text{sc}} - \tau_{\text{r}}) \right\} & (40)
\end{aligned}$$

The desired result is

$$I_{\text{det}} = \frac{1}{2}(1 + a_{\text{sc}}^2)\Gamma(0) + a_{\text{sc}} \text{Re}\left\{ \Gamma\left( 2 \cdot \frac{l_{\text{sc}} - l_{\text{r}} + \bar{n}z}{c} \right) \right\} \tag{41}$$

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**b)** According to the Wiener-Khintchine theorem discussed in script and lecture, the autocorrelation function is the Fourier transform of the spectral density $S(\omega) = |E(\omega)|^2$:

$$\Gamma(\tau) = \int_{-\infty}^{\infty} \mathrm{d}\omega S(\omega) \exp(-\mathrm{i}\omega\tau) \tag{42}$$

The spectral density was given in the problem to be a Gaussian with width $\Delta\omega$, centered around $\omega_0$:

$$S(\omega) = i_0 \exp\Big( - \frac{(\omega - \omega_0)^2}{2\Delta\omega^2}\Big) \tag{43}$$

We can use the Fourier table of Bronstein (there is a typo in edition 4, this is the correct version):

$$f(\omega) = \frac{1}{\sqrt{4\pi a}} \mathrm{e}^{-\omega^2/4a} \Rightarrow \mathrm{e}^{-a\tau^2} = \int_{-\infty}^{\infty} \mathrm{d}\tau\, f(\omega) \mathrm{e}^{-\mathrm{i}\omega t}, \tag{44}$$

or, equivalent but more convenient for us here (and I checked it), english Wikipedia:

$$f(\omega) = \mathrm{e}^{-\alpha\omega^2} \Rightarrow \sqrt{\frac{\pi}{\alpha}} \mathrm{e}^{-\tau^2/4\alpha} = \int_{-\infty}^{\infty} \mathrm{d}\tau\, f(\omega) \mathrm{e}^{-\mathrm{i}\omega\tau}. \tag{45}$$

We enter $S(\omega)$ into the Fourier integral and get

$$\Gamma(\tau) = \int_{-\infty}^{\infty} \mathrm{d}\omega\, i_0 \exp\Big( - \frac{(\omega - \omega_0)^2}{2\Delta\omega^2}\Big) \exp(-\mathrm{i}\omega\tau) \tag{46}$$

Using the substitution of variables $\omega' = \omega - \omega_0 \Rightarrow \mathrm{d}\omega' = \mathrm{d}\omega$ we obtain

$$\Gamma(\tau) = i_0 \exp(-\mathrm{i}\omega_0\tau) \int_{-\infty}^{\infty} \mathrm{d}\omega'\, \exp\Big( - \frac{\omega'^2}{2\Delta\omega^2}\Big) \exp(-\mathrm{i}\omega'\tau) \tag{47}$$

In this form, we can apply (45) and identify $\alpha = \frac{1}{2\Delta\omega^2}$, wich finally gives

$$\Gamma(\tau) = \sqrt{2\pi} i_0 \Delta\omega \exp(-\mathrm{i}\omega_0\tau) \exp(-\frac{1}{2}\Delta\omega^2\tau^2). \tag{48}$$

The final desired result is

$$|\Gamma(\tau)| = \sqrt{2\pi} i_0 \Delta\omega \exp\Big( - \frac{\tau^2\Delta\omega^2}{2}\Big). \tag{49}$$

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**c)** In terms of $s$, we find that $\tau = \tau_r - \tau_l = (s - 2\bar{n}z)/c$, hence

$$I_{\mathrm{det}} = \frac{1}{2}(1 + a_{\mathrm{sc}}^2)\Gamma(0) + a_{\mathrm{sc}} \mathrm{Re}\Big\{\Gamma\Big(\frac{s - 2\bar{n}z}{c}\Big)\Big\} \tag{50}$$

Inserting the explicit form for $\Gamma(\tau)$ from b) yields

$$I_{\mathrm{det}} = \sqrt{2\pi} i_0 \Delta\omega \Big[\frac{1}{2}(1 + a_{\mathrm{sc}}^2) + a_{\mathrm{sc}} \mathrm{Re}\Big\{ \exp\Big( - \frac{(s - 2\bar{n}z)^2\Delta\omega^2)}{2c^2}\Big) \cdot \exp\Big( - \mathrm{i}\omega_0 \frac{(s - 2\bar{n}z)}{c}\Big)\Big\}\Big] \tag{51}$$

$$= \sqrt{2\pi} i_0 \Delta\omega \Big[\frac{1}{2}(1 + a_{\mathrm{sc}}^2) + a_{\mathrm{sc}} \exp\Big( - \frac{\Delta\omega^2}{2c^2}(s - 2\bar{n}z)^2\Big) \cdot \cos\Big(\frac{2\omega_0}{c}(s - 2\bar{n}z)\Big)\Big] \tag{52}$$

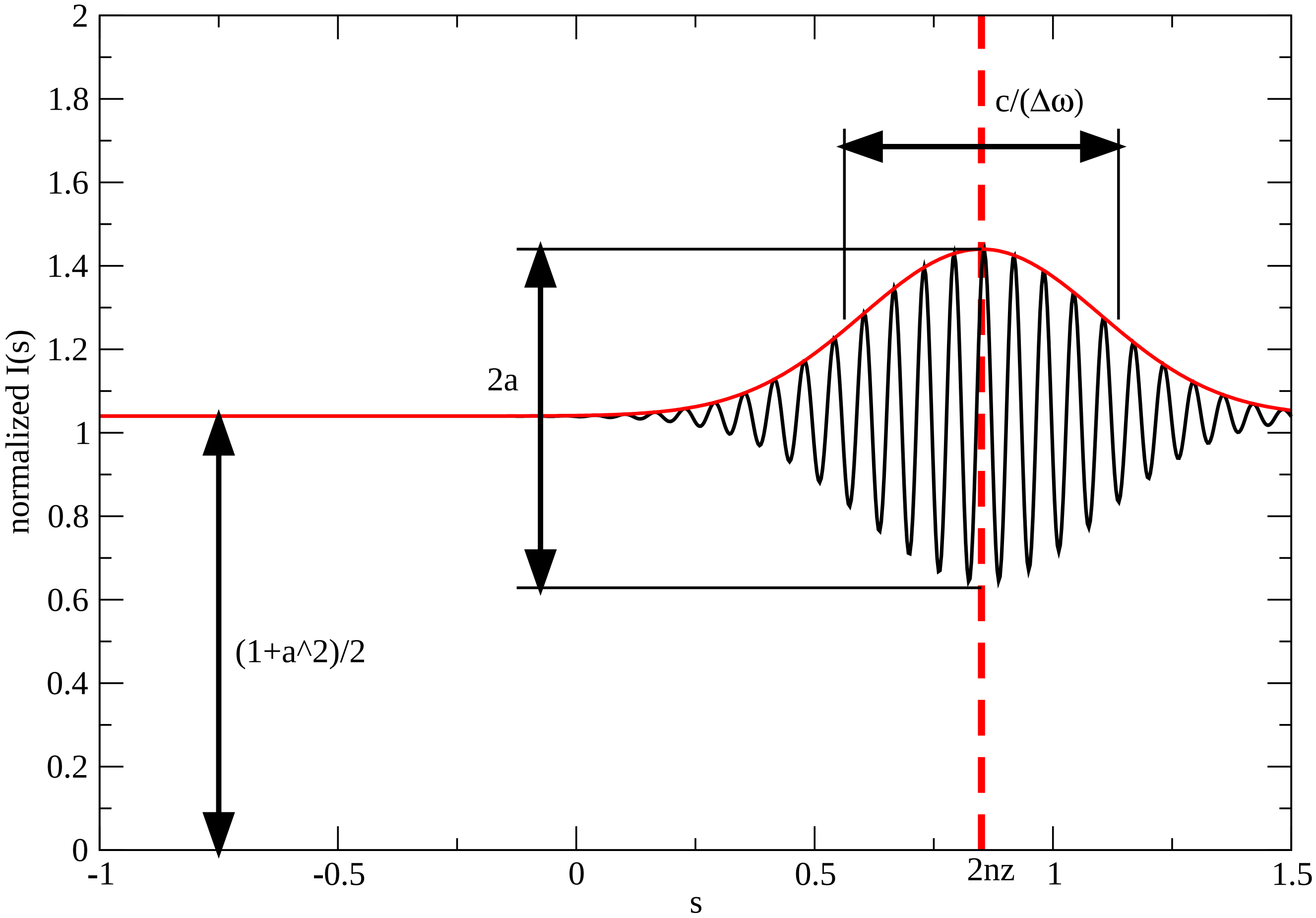


Figure 2: Fringe pattern corresponding to part c), with normalized $I(s) = I_{\text{det}}(s)/\sqrt{2\pi} i_0 \Delta\omega$.

The sketch (cf. Fig. 2) of $I_{\text{det}}$ as a function of $s$ shows a constant intensity of $\sqrt{2\pi} i_0 \Delta\omega(1 + a_{\text{sc}}^2)$ for $|s| \gg 0$. Around the position $s = 2\bar{n}z$, there is a fringe pattern with amplitude $\sqrt{2\pi} i_0 \Delta\omega 2a_{\text{sc}}$ on top of the constant intensity. The fringe pattern has a Gaussian envelope and decays with a spread of $\frac{c}{\Delta\omega}$.

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**d)** The depth $z_{\text{sc}}$ of the scatterer is encoded as the maximum of the Gaussian fringe envelope in the $I_{\text{det}}/s$–plot. By $s_{\max} = 2\bar{n}z$, we need the mean refractive index $\bar{n}$ of the sample, the distance between coupler and sample surface $l_{\text{s}}$ and the length of the reference $l_{\text{r}}$ arm and the intensity amplitude $i_0$ in order to determine $z_{\text{sc}}$ and $a_{\text{sc}}$ quantitatively.

The $z$-resolution is illustrated in Fig. 3. The scattering efficiency $a_{\text{sc}}$ enters the detected intensity as part of the background intensity and as the amplitude of the fringe pattern. The first occurence does not provide any information because the contributions of scatterers at different vertical positions in the sample are indistinguishable. Thus, the fringe amplitude defines the scattering amplitude as long as the fringe patterns of adjacent scatterers do not overlap (in this case, the problem becomes much harder).

A narrow fringe pattern is desirable because then scatteres can be closer together without having overlapping fringe patterns. The width of the fringe pattern is the width of the Gaussian envelope and is reciprocal to the spectral width. This means

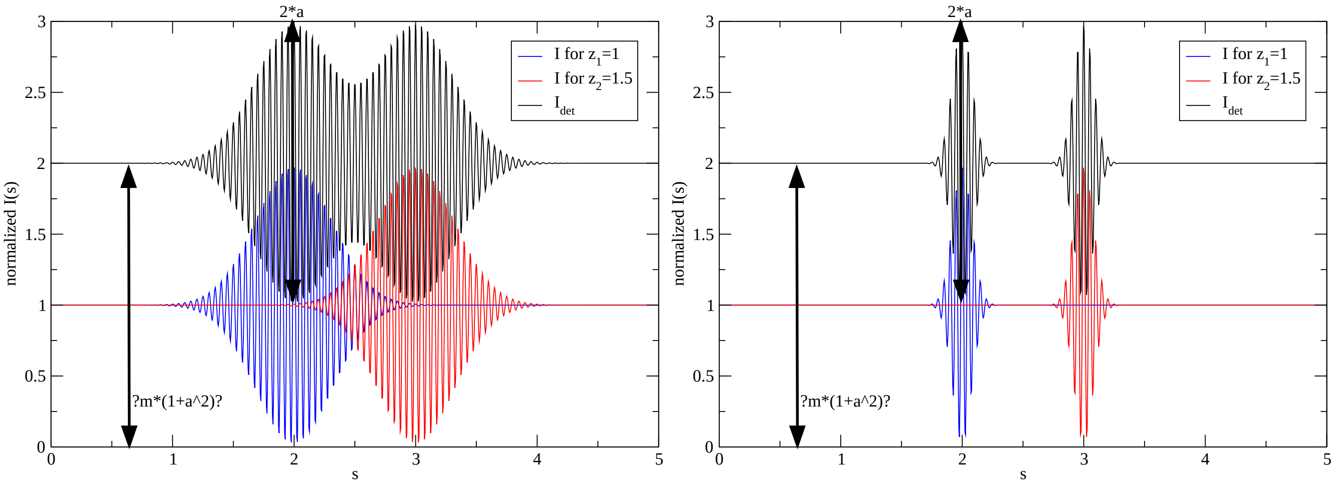


Figure 3: Naive schematic of superposed intensities of two scatterers at $z_1 = 1$ and $z_2 = 1.5$ (interference effects have been neglected, but the basic behavior is visible). Left: small $\Delta\omega$, corresponds to high degree coherent light. Right: large $\Delta\omega$, corresponds to low degree of coherence. The "zero-line" of the intensity of height $m(1+a_{\rm sc}^2)/2$ is no good measure for $a_{\rm sc}$, since an unkown number $m$ of scatterers contribute to this value. The Gaussian envelope however is always of width $2a_{\rm sc}$, no matter how many scatterers contribute to the signal. Left: High coherence causes reflected signals to overlap and reduce the $z$-resolution. Right: Low coherence narrows the fringes and $z$-positions of individual scatterers are better discernible. This allows better $z$-resolution than conventional microscopy.

that a *reduced* coherence of the light source improves the vertical resolution.

In the lateral direction, the spectrometer behaves as a conventional microscope and, thus, it is diffraction-limited. In particular, there is no dependence on the coherence properties of the light source.

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— Hand in solutions in lecture on —