Karlsruher Institut für Technologie (KIT) SS2012
Institut für theoretische Festkörperphysik
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Tutorial:

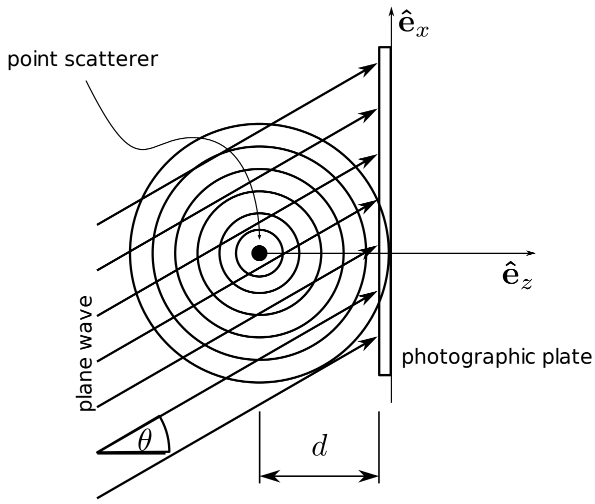
- [ ] **Group 1,**
- [ ] **Group 2,**
- [ ] **Group 3.**
- [ ] **Group 4.**

Name:____

Problem set 6 for the course "Theoretical Optics"

12 Hologram Of A Point Scatterer

A point scatterer resides on the z-axis in distance d to a photographic plate at $z = 0$. A plane reference wave $\Psi_{\text{ref}}(\mathbf{r}) = A_{\text{ref}} e^{-ik_0 x \sin\theta}$ interferes on the screen with the object wave $\Psi_{\text{obj}}(x,y) = \frac{A_{\text{obj}}}{\sqrt{x^2+y^2+d^2}} e^{-ik_0\sqrt{x^2+y^2+d^2}}$, where $A_{\text{ref}}, A_{\text{obj}} \in \mathbb{R}$. We consider a large distance d between object and screen.



a) We want to calculate the interference effects on the screen in the following. Therefore, find an approximate expression of the object wave Ψ_{obj} for large distance d between object and screen. Explain, what exactly this property means mathematically. Explain, why this allows You to approximate the square roots by Taylor polynomials then and why it suffices to expand the amplitude up to zeroth order in x and y (means that the approximation is independent of x and y), while for the argument of the exponential the next non-vanishing order of the variables x and y has to be included. [**2** Point(s)]

b) The photographic screen will show the interference pattern of the reference and object wave after developing, thus acting as an aperture with amplitude transmission coefficient $T(x,y) = |\Psi_{\text{obj}} + \Psi_{\text{ref}}|^2$. Show that

$$T(x,y) = A_{\text{ref}}^2 + \frac{A_{\text{obj}}^2}{d^2} + 2\frac{A_{\text{ref}}A_{\text{obj}}}{d} \cos\left(k_0 d - \frac{k_0 d \sin^2\theta}{2} + \frac{k_0}{2d}\left((x - d\sin\theta)^2 + y^2\right)\right). \quad (1)$$

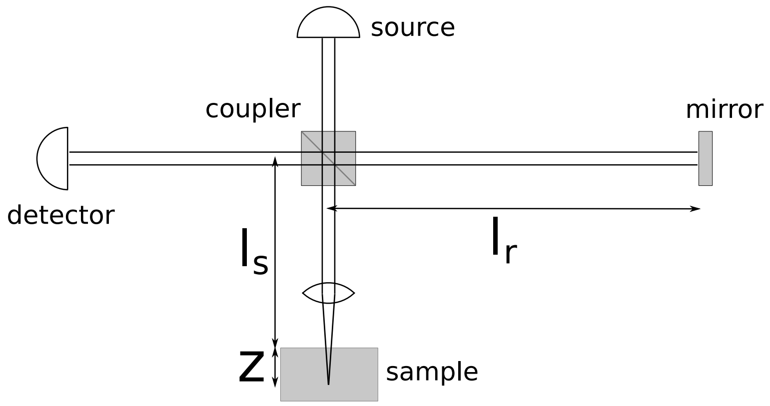
[**3** Point(s)]

c) To reconstruct the image of the point scatterer, we illuminate the developed screen with an incoming plane wave $\Psi_{\text{in}} = \Psi_{\text{ref}}$ of the same frequency as the reference wave. Directly behind the screen, the transmitted field distribution is then given by $\Psi_{\text{trans}} = \Psi_{\text{in}} T(x,y)$. Find the field $\Psi_{\text{trans}}(x,y)$ on the screen for general θ in terms of propagating waves (express sin and cos by complex exponential functions). Which of these partial waves reconstructs the image of the point scatterer? What is the physical meaning of the remaining partial waves?[**3** Point(s)]

13 Optical Coherence Tomography

Optical coherence tomography is a scanning optical microscope with improved vertical resolution.

From a source, light with amplitude $E_0(t)$ emerges and is split into a reference ray and a measurement ray. This is focused by a conventional microscope into a sample with mean refractive index $\bar{n}$. $E_{\text{r}}(t)$ is the amplitude of the reflected reference ray at the position of the coupler; $E_{\text{sc}}(z;t)$ is the amplitude of the light scattered inside the sample at the position of the coupler. The light is scattered at some particle located at depth z with the real–valued scattering efficiency a_{sc}.



a) Formulate $E_{\text{r}}(t)$ and $E_{\text{sc}}(z;t)$ in terms of $E_0(t)$. Express the detected intensity $I_{\text{det}} = \left\langle \left| E_{\text{r}}(t) + E_{\text{sc}}(t) \right|^2 \right\rangle$ in terms of the autocorrelation function $\Gamma(\tau) = \langle E_0(t) E_0^*(t-\tau) \rangle$.[**4** Point(s)]

b) Assume that E_0 has a Gaussian spectral power density with mean angular frequency ω_0 and width $\Delta\omega$:

$$\left| E_0(\omega) \right|^2 = i_0 \cdot \exp\left(-\frac{(\omega - \omega_0)^2}{2\Delta\omega^2} \right) \quad (2)$$

Find $\left| \Gamma(\tau) \right|$. [**3** Point(s)]

c) Consider a single point scatterer at $z = z_{\text{sc}}$. Calculate I_{det} as a function of $s := 2l_r - 2l_{\text{s}}$ using the results of **a)** and **b)**. Sketch $I_{\text{det}}(s)$ and annotate the main features. [**3** Point(s)]

d) Which features of the interferometer and of $I_{\text{det}}(s)$ have to be known to determine the quantities z_{sc} and a_{sc}? How does the light source influence the vertical resolution (z-direction)? What determines the lateral resolution (x-direction)?[**2** Point(s)]

— Hand in solutions in tutorial on 25.06.2012 —