Karlsruher Institut für Technologie (KIT) SS2012
Institut für theoretische Festkörperphysik
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Tutorial:

☐ **Group 1,**

☐ **Group 2,**

☐ **Group 3.**

☐ **Group 4.**

Name:____

Problem set 5 for the course "Theoretical Optics"

10 **Fresnel Diffraction by a Slit**

We consider a monochromatic plane wave impinging normally on an opaque screen with a slit (of width w and height h) in it (Fig. 1).

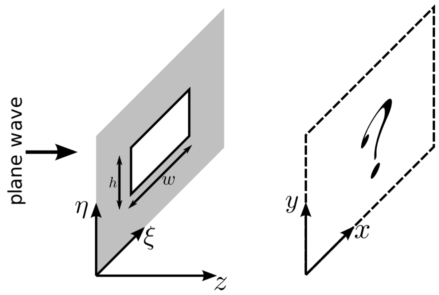

Figure 1: A rectangular shaped aperture.

a) Starting from the Fresnel diffraction integral expressed as

$$U(\mathbf{r}) = \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z} \iint_{-\infty}^{\infty} \mathrm{d}\xi \mathrm{d}\eta \, U(\xi,\eta) \, \mathrm{e}^{\mathrm{i}\frac{k}{2z}[(x-\xi)^2+(y-\eta)^2]} \tag{1}$$

with $k = \frac{2\pi}{\lambda}$, put the origin of the coordinate system into the center of the rectangular opening and enter the slit function $U(\xi,\eta)$. Then, normalize all distances to make the expression independent of the wavelength λ. Write the integral as a product of two independent integrals. Please state your substitutions explicitly. [**3** Point(s)]

b) Reformulate the integral from a) in terms of the so-called Fresnel integrals given by

$$C(t) = \int_0^t \cos\left(\frac{\pi}{2}x^2\right) \mathrm{d}x, \tag{2}$$

$$S(t) = \int_0^t \sin\left(\frac{\pi}{2}x^2\right) \mathrm{d}x. \tag{3}$$

Finally, calculate the intensity $|U(x,y)|^2$ at distance z in terms of C and S. Again, please provide the individual steps leading to your final result. [**5** Point(s)]

c) Use a suitable computer program which is able to evaluate Fresnel integrals (e.g. Mathematica or Maple) to plot the intensity distribution for different aperture sizes and distances. Consider the following cases: $w = h = 1\lambda$, $w = h = 2\lambda$, $w = h = 10\lambda$ and $w = 1\lambda, h = 2\lambda$. For the distances, use $z = 1\lambda,\ 10\lambda,\ 100\lambda,\ 1000\lambda,\ 10^6\lambda$ and $10^9\lambda$. [**2 BONUS** Point(s)]

11 Fraunhofer Diffraction by a Slit

We consider the same system as in the previous problem, but now using the Fraunhofer approximation.

a) Write down the scalar field on the screen in the Fraunhofer approximation. As before, choose the center of the rectangular slit as the origin, substitute the functions U and make the expression independent of the wavelength λ. Then, perform the integration to obtain the field and the intensity distributions at distance z. Express Your solutions in terms of the unnormalized sinc function $\mathrm{sinc}(x) := \sin(x)/x$.[**4** Point(s)]

b) Again, use a suitable computer program to plot the intensity distributions for the same parameters as given in 12c).[**2 BONUS** Point(s)]

— Hand in solutions in tutorial on 11.06.2011 —