Karlsruher Institut für Technologie (KIT) SS2012
Institut für theoretische Festkörperphysik
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Tutorial:

☐ **Group 1,**

☐ **Group 2,**

☐ **Group 3.**

☐ **Group 4.**

Name:______________________________

Problem set 4 for the course "Theoretical Optics"

7 Light Propagation in Anisotropic Media

The dispersion relation in a general anisotropic medium is given by the Fresnel equation

$$\left(\frac{c^2k^2}{\omega^2}\right)\left(\frac{\epsilon_x c^2 k_x^2}{\omega^2}+\frac{\epsilon_y c^2 k_y^2}{\omega^2}+\frac{\epsilon_z c^2 k_z^2}{\omega^2}\right) - \left(\frac{c^2k_x^2}{\omega^2}\epsilon_x\left(\epsilon_y+\epsilon_z\right)+\frac{c^2k_y^2}{\omega^2}\epsilon_y\left(\epsilon_x+\epsilon_z\right)+\frac{c^2k_z^2}{\omega^2}\epsilon_z\left(\epsilon_x+\epsilon_y\right)\right) + \epsilon_x\epsilon_y\epsilon_z = 0. \quad (1)$$

a) Show that the Fresnel equation can be recast into the form

$$s_x^2\left(v_p^2-v_y^2\right)\left(v_p^2-v_z^2\right)+s_y^2\left(v_p^2-v_x^2\right)\left(v_p^2-v_z^2\right)+s_z^2\left(v_p^2-v_x^2\right)\left(v_p^2-v_y^2\right)=0. \quad (2)$$

Here, we introduced the phase velocity $v_p = \frac{\omega}{|\mathbf{k}|}$ of the wave, the phase velocities along the coordinate axes $v_i = \frac{c}{\sqrt{\epsilon_i}}$ and the normalized wave vector components $s_i = \frac{k_i}{|\mathbf{k}|}$. [**4** Point(s)]

b) Now, we want to demonstrate that, in general, there are two solutions (phase velocities) for every given propagation direction $\mathbf{s} = (s_x, s_y, s_z)$. To find these solutions, assume that $\epsilon_x < \epsilon_y < \epsilon_z$ and insert

$$v_x^2 = v_y^2 + q_x, \quad v_z^2 = v_y^2 - q_z, \quad v_p^2 = v_y^2 + q \quad (3)$$

into the Fresnel equation (2). Calculate the two possible solutions q', q'' for q and show that they must have opposite sign, i.e., $q' \cdot q'' \leq 0$. Here, we choose $q' \geq 0$. Show that then, the following inequality holds:

$$-q_z \leq q'' \leq 0 \leq q' \leq q_x. \quad (4)$$

[**4** Point(s)]

c) There are two distinct directions, where only one solution exists, so $q' = q''$. These directions form the two optical axes of the crystal. Show that these optical axes must lie in the (x, z)-plane and demonstrate that the angle β which the z-axis encloses with the two axes is given by

$$\tan\beta = \pm\sqrt{\frac{v_x^2 - v_y^2}{v_y^2 - v_z^2}}. \tag{5}$$

[**3** Point(s)]

8 Reflection at Faraday Rotators

We consider the reflection of a normally incident linearly polarized plane wave

$$\mathbf{E}_\mathrm{i}(z,t) = E_\mathrm{i}\hat{\mathbf{e}}_x e^{\mathrm{i}(kz-\omega t)} \tag{6}$$

at an air-material interface. We want to determine the polarization and intensity of the reflected wave.

Inside the material, the transmitted wave propagates as two circularly polarized plane waves with refractive indices n_+ and n_- as

$$\mathbf{E}_\mathrm{t} = E_{\mathrm{t},+}\begin{pmatrix}1\\+\mathrm{i}\end{pmatrix}\mathrm{e}^{\mathrm{i}(k_+z-\omega t)} + E_{\mathrm{t},-}\begin{pmatrix}1\\-\mathrm{i}\end{pmatrix}\mathrm{e}^{\mathrm{i}(k_-z-\omega t)}, \tag{7}$$

with the two dispersion relations

$$k_\pm = n_\pm\omega/c. \tag{8}$$

For the electric field amplitudes we assume $E_\mathrm{i}, E_{\mathrm{t},\pm} \in \mathbb{C}$.

a) Consider the incoming linearly polarized wave as a superposition of circularly polarized waves (compare Prob. 5). Show, that the incoming circular parts can only excite transmitted/reflected circular parts of the same polarization orientation. [**1** Point(s)]

b) Use the result from a) to determine the amplitude of the reflected waves in terms of $n_\pm$ for each polarization state independently. *Hint:* Use the continuity conditions for the E- and H-field components to derive a set of equations relating E_t and E_r to E_i (transmitted, incoming and reflected amplitude), incorporating the respective indices of refraction. Be aware of phase jumps and use $\mu = \mu_0$ (a commonly used approximation for optical frequencies). [**1.5** Point(s)]

c) Show that the full reflected wave fulfills

$$\frac{|\mathbf{E}_\mathrm{r}|^2}{|\mathbf{E}_\mathrm{i}|^2} = \frac{1}{2}\left[\left(\frac{1-n_+}{1+n_+}\right)^2 + \left(\frac{1-n_-}{1+n_-}\right)^2\right]. \tag{9}$$

[**1** Point(s)]

d) What is the polarization state of the reflected wave? [**0.5** Point(s)]

— Hand in solutions in tutorial on 04.06.2012 —