Karlsruher Institut für Technologie (KIT) SS2012
Institut für theoretische Festkörperphysik
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Tutorial:

☐ **Group 1,**

☐ **Group 2,**

☐ **Group 3.**

☐ **Group 4.**

Name:____

Problem set 3 for the course "Theoretical Optics"

5 Polarization States Of Plane Waves

We want to investigate superposition states of different kinds of polarizations.

a) We consider two circularly polarized plane waves with opposite sense of rotation at $\mathbf{r} = 0$ ($E_0 \in \mathbb{R}$):

$$\begin{aligned} \mathbf{E}_1(\mathbf{r} = 0, t) &= E_0\big(\hat{\mathbf{e}}_x \cos(\omega t) + \hat{\mathbf{e}}_y \sin(\omega t)\big) \\ \mathbf{E}_2(\mathbf{r} = 0, t) &= E_0\big(\hat{\mathbf{e}}_x \cos(\omega t + \phi) - \hat{\mathbf{e}}_y \sin(\omega t + \phi)\big) \end{aligned} \tag{1}$$

Show that at $\mathbf{r} = 0$ the superposition $\mathbf{E}(t) = \mathbf{E}_1(t) + \mathbf{E}_2(t)$ is a linearly polarized plane wave. [**3** Point(s)]

b) Show that an elliptically polarized plane wave at $\mathbf{r} = 0$,

$$\mathbf{E}(\mathbf{r} = 0, t) = \hat{\mathbf{e}}_x A \cos(\omega t) + \hat{\mathbf{e}}_y B \sin(\omega t) \quad \text{with } A, B \in \mathbb{R}, \tag{2}$$

can be written as the superposition of a circularly polarized plane wave and a linearly polarized plane wave (also at $\mathbf{r} = 0$). [**2** Point(s)]

6 Optical Tweezer [6 extra points]

The optical tweezer [L. Novotny et al., PRL, **79**, 4, 645] is one of the best examples for the application of the Maxwell Stress Tensor. We consider the force, which acts on a particle placed within a laser beam. For simplicity approximate the laser by a so-called Gaussian beam, for many experimental situations this is a reasonable well approximation.

The electric field of a Gaussian beam is given in terms the radius r (radial distance from the center axis of the beam) and z (axial distance from the beam's narrowest point) in cylinder coordinates by the following expression:

$$E(r, z) = E_0 \frac{w_0}{w(z)} \exp\left(-\left(r/w(z)\right)^2\right) \exp\left(-ik\frac{r^2}{2R(z)}\right) \exp\left(i\left(\xi(z) - kz\right)\right)$$

where

$$\begin{aligned}
w_0 &- \text{minimum beam radius} \\
w(z) &= w_0\sqrt{1+\left(\frac{z}{z_0}\right)^2} - \text{beam radius} \\
z_0 &= \frac{\pi w_0^2}{\lambda} \\
R(z) &= z\left(1+\left(\frac{z_0}{z}\right)^2\right) \\
\xi(z) &= \arctan\left(\frac{z}{z_0}\right) \\
k &= \frac{2\pi}{\lambda}
\end{aligned}$$

Given the electric field, a simple way to approximate the force on a small particle (electric field assumed homogeneous over the size of the particle) is given by

$$\vec{F} = \frac{\alpha}{2}\vec{\nabla}E^2$$

where α is the particle's polarizability.

(i) Numerically calculate the force component in radial direction and visualize your results in a plot. Argue how your result explains the action of an optical tweezer.

A rigorous treatment, beyond the Rayleigh limitations above, yields the following expression for the force $\vec{F}$

$$\vec{F} = \int_{\partial V} \langle T \cdot \vec{n} \rangle \mathrm{d}S$$

where $\langle \ldots \rangle$ denotes a time average and T is the (electric) Maxwell Stress Tensor given by

$$T = \epsilon_0 \epsilon \vec{E}\vec{E} - \frac{1}{2}\epsilon_0 \epsilon E^2$$

(ii) For the above expression of the force, repeat your considerations from (i).

— Hand in solutions in tutorial on 21.05.2012 —