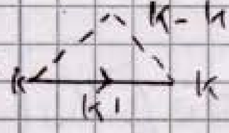
Aufgabe 1a)

$\Sigma^{(1)} =$ 

$$= N_i \sum_{\vec{k}'} |v(\vec{k}-\vec{k}')|^2 \, G(\vec{k}', i\omega_n)$$

$$= N_i \, v_0^2 \sum_{\vec{k}'} \frac{1}{i\omega_n - \epsilon_{k'}}$$

$$= N_i \, v_0^2 \int \frac{d^3\vec{k}'}{4\pi} \, \frac{1}{i\omega_n - \epsilon_{k'}}$$

$$= N_i \, v_0^2 \int_0^\infty dk \, \frac{k^2}{i\omega_n - \epsilon_k}$$

Mit der Zustandsdichte $N(E)$ ergibt sich

$$k^2 \, dk = N(E) \, dE$$

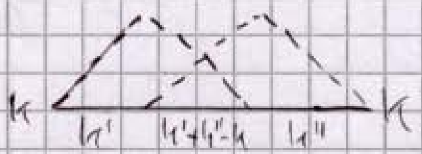
um die Fermikante

$$= N_i \, N_0(0) \, v_0^2 \int_{-\infty}^{\infty} dE \, \frac{1}{i\omega_n - \epsilon_k}$$

$$= -i N_i \, N(0) \, v_0^2 \sin 2\pi \; \mathrm{sign}(\omega_n)$$

$$= -i \frac{1}{2\tau} \, \mathrm{sign}(\omega_n)$$

Aufgabe 1b)

$$\Sigma^{(2)} =$$ 

$$= |N_i|^2 \sum_{k'} \sum_{k''} |V(k-k')|^2 |V(k-k'')|^2 G(k') G(k'+k''-k) G(k'')$$

$$= N_i^2 v_0^4 \sum_{k'} \sum_{k''} G(k') G(k'') G(k'+k''-k)$$

$$= N_i^2 v_0^4 \int \frac{d^3\vec{k}_1}{4\pi} \int \frac{d^3\vec{k}_2}{4\pi} G(\vec{k}_1) G(\vec{k}_2) G(\vec{k}_1+\vec{k}_2-\vec{k})$$

$$= N_i^2 v_0^4 \int dk_1 \frac{k_1^2}{i\omega_n - \epsilon_{\vec{k}_1}} \int dk_2 \frac{k_2^2}{i\omega_n - \epsilon_{\vec{k}_2}} \int \frac{d\Omega_1 d\Omega_2}{(4\pi)^2} \frac{1}{i\omega_n - \epsilon_{\vec{k}_1+\vec{k}_2-\vec{k}}}$$

da $\vec{k} \approx k_F$ folgt $\vec{k}_1 \approx -\vec{k}_2$

damit entwickeln wir:

$$\epsilon_{\vec{k}_1+\vec{k}_2-\vec{k}} \approx \epsilon_{-\vec{k}} - \vec{v}_F(\vec{k}_1+\vec{k}_2) = \epsilon_{-\vec{k}} - v_F(k_1\cos\Theta_1 + k_2\cos\Theta_2)$$

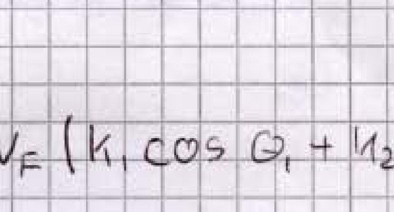
$$v_F = \nabla_{\vec{k}} \epsilon_{+\vec{k}}$$

$$\int \frac{d\Omega_1 d\Omega_2}{(4\pi)^2} \frac{1}{i\omega_n - \epsilon_{\vec{k}_1+\vec{k}_2-\vec{k}}} = \frac{1}{4} \int_{-1}^{1} d\cos\Theta_1 \int_{-1}^{1} d\cos\Theta_2 \frac{1}{i\omega_n - \epsilon_{-\vec{k}} - v_F(k_1\cos\Theta_1 + k_2\cos\Theta_2)}$$

$k_1 \approx k_2 \approx k_F$ new variables $\cos\Theta_1 + \cos\Theta_2 = X$

$\cos\Theta_1 - \cos\Theta_2 = Y$

$$\frac{\partial(x,y)}{\partial(\cos\Theta_1, \cos\Theta_2)} = -2$$

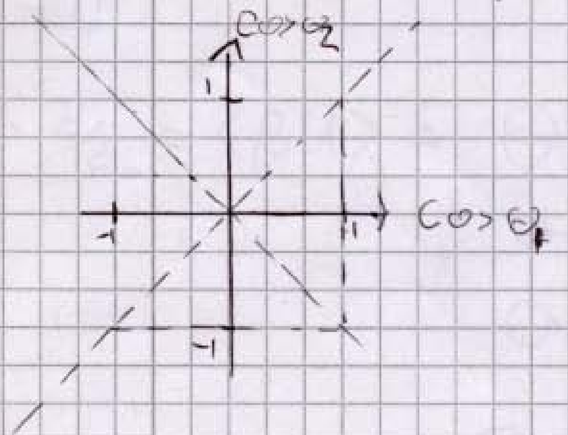
Integration: 

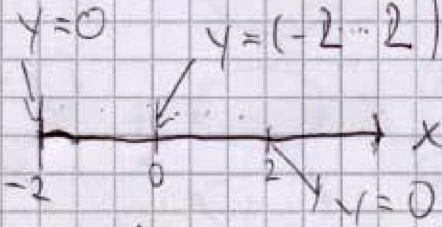
$$\approx -\frac{1}{8} \int dX \int dY \frac{1}{i\omega_n - \epsilon_{-\vec{k}} - v_F\left[k_1((X+Y)/2) + k_2((X-Y)/2)\right]}$$

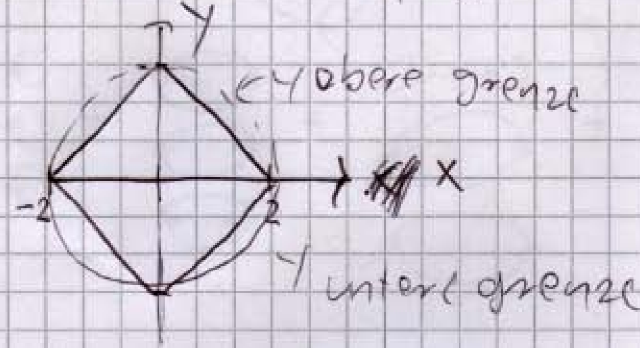
$$= \underline{I}$$

Koordinaten: $\cos\Theta_1 + \cos\Theta_2 = x$

$\cos\Theta_1 - \cos\Theta_2 = y$







$$Y_{up} = 2 - |x|$$

$$Y_{down} = -2 + |x|$$

$$I \approx -\frac{1}{8}\int_{-2}^{2} dx \int_{-2+|x|}^{2-|x|} dy \, \frac{1}{i\omega_n - \epsilon_{-\vec{k}} - v_F k_F x}$$

$$= -\frac{1}{4}\int_{-2}^{2} dx \, \frac{2-|x|}{i\omega_n - \epsilon_{-\vec{k}} - v_F k_F x}$$

Residuum: $v_F k_F x = i\omega_n - \epsilon_{-\vec{k}}$

wir nehmen an $|x| = \left|\frac{i\omega_n - \epsilon_{-\vec{k}}}{v_F k_F}\right| \propto 1$

$v_F k_F = E_F \; \bar{V}_0$

alle Energie nahe der Fermi energie.

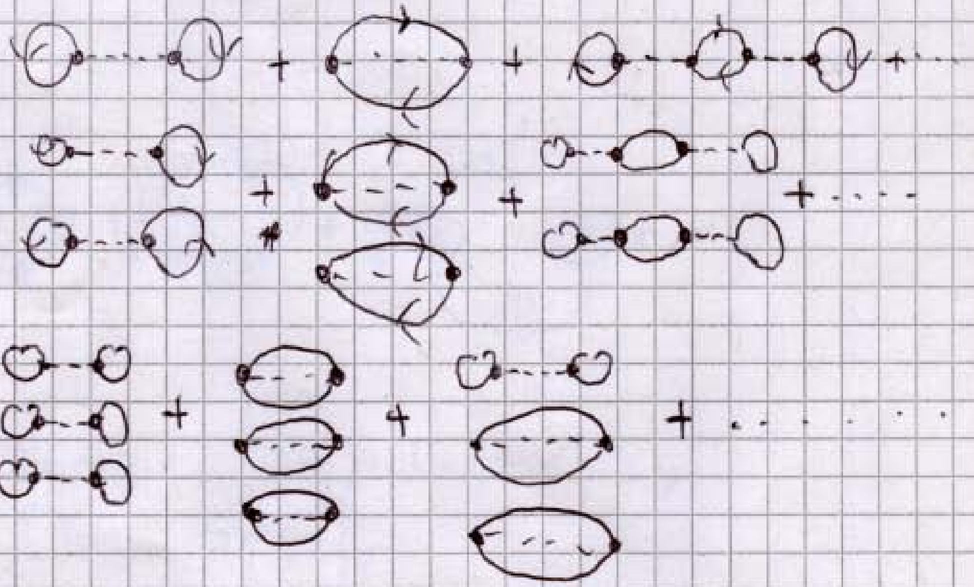
$$\Rightarrow \quad I \propto \frac{1}{v_F k_F}$$

$$\Sigma^{(2)} \propto \frac{1}{J}\,\frac{1}{J}\,\frac{1}{v_F k_F} \propto \frac{1}{J}\,\frac{1}{k_F l}$$

Aufgabe A 2a)

c

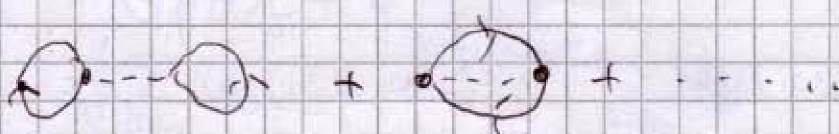
$\frac{Z}{Z_0} = \langle \beta U \rangle =$



$= U_1 + \frac{1}{2!} U_1^2 + \frac{1}{3!} U_1^3$

$= e^{U_1}$

$U_1 =$



detailierte Herleitung:

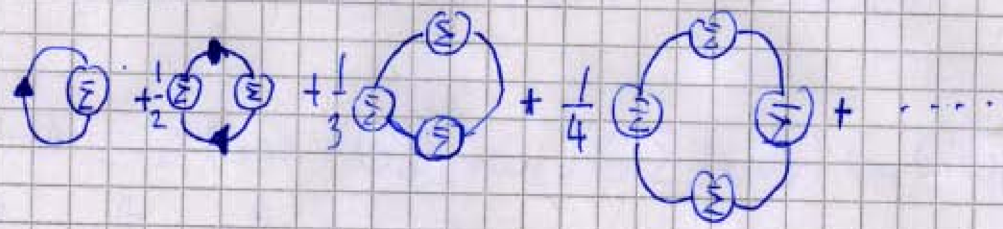
"Many-Particle Physics"

Gerald D. Mahan 3.6. A

beginnend bei Gleichung 3.6.7

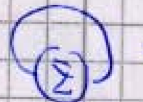
Aufgabe 2b)

Gegeben:

$\ln Z/Z_0 =$  $+ \cdots\cdots$

Wichtig

$\xrightarrow[\omega_n, q]{} \Sigma \xrightarrow[\omega_n, q]{}$ Energie, Impulserhaltung [kein Spin]

 $= \underset{\text{Ein Loop}}{(-1)} \sum_q \sum_{\omega_n} G_0(i\omega_n, q)\, \Sigma(i\omega_n, q)$

Test  $\propto G(\tau_2, \tau_1)\, \Sigma(\tau_1, \tau_2)$ $\quad \Delta\tau = \tau_2 - \tau_1$ $\quad \cancel{\Delta\tau = \tau_2 - \tau_1}$

$$= \frac{1}{\beta^2} \sum_{i\omega_n} G_0(i\omega_n, q)\, e^{-i\omega_n \Delta\tau} \sum_{i\omega_m} \Sigma(i\omega_m, q)\, e^{+i\omega_m \Delta\tau}$$

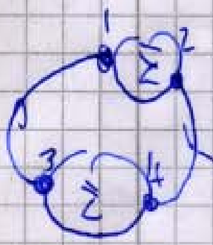
$$\cancel{= \frac{1}{\beta} \sum_{i\omega_m} G_0\, e^{i\omega_n \tau}}$$

$$\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \; e^{-i(\omega_n - \omega_m)\tau_2}\, e^{i(\omega_n - \omega_m)\tau_1}$$

$$= \frac{1}{(-i(\omega_n - \omega_m))} \left[e^{-i(\omega_n - \omega_m)\tau_2}\right]_0^\beta \left[e^{i(\omega_n - \omega_m)\tau_1}\right]_0^\beta \frac{1}{i(\omega_n - \omega_m)}$$

$$= \begin{cases} 0 & \text{für } \omega_n \neq \omega_m \\ \beta^2 & \text{für } \omega_n = \omega_m \end{cases}$$

Test II:



$$\propto \Sigma(2,1)\, G_0(4,2)\, \Sigma(3,4)\, G_0(1,3)$$

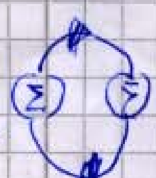
$$\longrightarrow \quad \Sigma(i\omega_1)\, G_0(i\omega_2)\, \cancel{G}\, \Sigma(i\omega_3)\, G_0(i\omega_4)$$

$$\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \int_0^\beta d\tau_3 \int_0^\beta d\tau_4 \; e^{-i(\omega_4-\omega_1)\tau_1}\, e^{-i(\omega_1-\omega_2)\tau_2}$$

$$\times e^{-i(\omega_3-\omega_4)\tau_3}\, e^{-i(\omega_2-\omega_3)\tau_4}$$

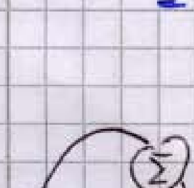
$$= \begin{cases} 0 & \text{für } \omega_4 \neq \omega_1 \,||\, \omega_1 \neq \omega_2 \,||\, \omega_3 \neq \omega_4 \,||\, \omega_2 \neq \omega_3 \\ \beta^4 & \text{für } \omega_1 = \omega_2 = \omega_3 = \omega_4 \end{cases}$$

usw.

$\Rightarrow \frac{1}{2}$ 

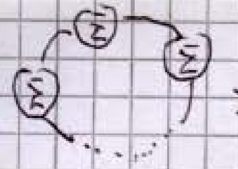
$$= (-1)\,\frac{1}{2} \sum_q \sum_{\omega_n} G_0(i\omega_n,q)\, \Sigma(i\omega_n,q)\, G_0(i\omega_n,q)\, \Sigma(i\omega_n,q)$$

$$= (-1)\,\frac{1}{2} \sum_q \sum_{\omega_n} G_0^2(i\omega_n,q)\, \Sigma^2(i\omega_n,q)$$

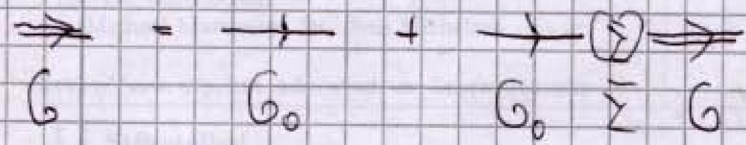
$\frac{1}{3}$ 

$$= (-1)\,\frac{1}{3} \sum_q \sum_{\omega_n} G_0(i\omega_n,q)\, \Sigma(i\omega_n,q)\, G_0(i\omega_n,q)\, \Sigma(i\omega_n,q)\, G_0(i\omega_n,q)\, \Sigma(i\omega_n,q)$$

$$= (-1)\,\frac{1}{3} \sum_{q,\omega_n} G_0^3(i\omega_n,q)\, \Sigma^3(i\omega_n,q)$$

$\frac{1}{n}$  $= (-1)\,\frac{1}{n}\sum_{q,\omega_n} G_0^n(i\omega_n,q)\,\Sigma^n(i\omega_n,q)$

Die Greensfunktion ist gegeben durch



$$\Rightarrow (1-G_0\Sigma)\,G = G_0 \qquad \Rightarrow (1-G_0\Sigma) = \frac{G_0}{G}$$

$$\ln Z/Z_0 = \sum_{m=1}^{\infty}(-1)\sum_{q,\omega_n}\frac{1}{m}\left[G_0\Sigma\right]^m$$

$$= \sum_{q,\omega_n}\ln\left[1-G_0(i\omega_n,q)\,\Sigma(i\omega_n,q)\right]$$

$$= -\sum_{q,\omega_n}\ln\left[\cancel{1+}\ \frac{G}{G_0}\right]$$

Hinweis: $\Delta\Omega = -T\ln Z/Z_0$

Minus fehlt auf Aufgabenblatt

$$\Rightarrow \Delta\Omega = \lim_{T\to 0} T\sum_{q,\omega_n} e^{i\omega_n J}\ln\left[\frac{G}{G_0}\right]$$