Blatt 1

Aufgabe 1 a)

$$P(\alpha, \alpha^*) = \frac{e^{|\alpha|^2}}{\pi^2} \int d^2\beta \, \langle -\beta | \rho | \beta \rangle \, e^{|\beta|^2} \, e^{-\beta\alpha^* + \beta^*\alpha} \, d^2\beta$$

$\langle -\beta \quad \rho = |n\rangle\langle n|$

$$\langle -\beta | \rho | \beta \rangle = e^{-|\beta|^2} \frac{(-1)^n |\beta|^{2n}}{n!}$$

$$P(\alpha, \alpha^*) = \frac{e^{|\alpha|^2}}{\pi^2 n!} \frac{\partial^{2n}}{\partial\alpha^n \partial\alpha^{*n}} \int d^2\beta \, e^{-\beta\alpha^* + \beta^*\alpha}$$

$$= \frac{e^{|\alpha|^2}}{n!} \frac{\partial^{2n}}{\partial\alpha^n \partial\alpha^{*n}} \delta^{(2)}(\alpha)$$

Aufgabe 1b)

$$W(\alpha,\alpha^*) = \frac{2}{\pi}\int d^2\lambda\, e^{-2|\lambda-\alpha|^2}\, P(\lambda,\lambda^*)$$

$$= \frac{2}{\pi}\frac{1}{n!} e^{-2|\alpha|^2} \frac{\partial^{2n}}{\partial\lambda^n\,\partial\lambda^{*n}} e^{-2|\lambda|^2} e^{2\lambda\alpha^*} e^{2\lambda^*\alpha}\Big|_{\lambda=\lambda^*=0}$$

$$\frac{\partial^{2n}}{\partial\lambda^n\,\partial\lambda^{*n}} e^{-A|\lambda|^2} e^{B\lambda} e^{C\lambda^*} = e^{B\lambda} e^{C\lambda^*}\left(B+\frac{\partial}{\partial\lambda}\right)^n\left(C+\frac{\partial}{\partial\lambda^*}\right)^n e^{-A|\lambda|^2}$$

$$= e^{B\lambda} e^{C\lambda}\left(B+\frac{\partial}{\partial\lambda}\right)^n (C-A\lambda)^n e^{-A|\lambda|^2}$$

$\left(B+\frac{\partial}{\partial\lambda}\right)^n (C-\lambda A)^n$ ~~see~~ (siehe nächste seite)

$$= \left(B+\frac{\partial}{\partial\lambda}\right)^{n-l} \sum_{k=0}^{l} (-1)^k \frac{l!}{k!(l-k)!}\frac{n!}{(n-k)!} A^k (C-\lambda A)^{n-k}$$

$$\times \left(B+\frac{\partial}{\partial\lambda}\right)^{n-k}$$

$$A = 2,\ B^* = C = 2\alpha$$

$$W(\alpha,\alpha^*) = \frac{2}{\pi}\frac{1}{n!} e^{-2|\alpha|^2} \sum_{k=0}^{n} (-1)^{n-k} \frac{n!}{(n-k)!k!}\frac{n!}{k!}$$

$$\times |2\alpha|^{2k}$$

$$\left(B+\frac{\partial}{\partial\lambda}\right)^{n-1}\left(B+\frac{\partial}{\partial\lambda}\right)(C-\lambda A)^{n}$$

$$=\left(B+\frac{\partial}{\partial\lambda}\right)^{n}\left[(C-\lambda A)^{n}\left(B+\frac{\partial}{\partial\lambda}\right)+(-1)\,n\,A(C-\lambda A)^{n-1}\right]$$

$$\left(B+\frac{\partial}{\partial\lambda}\right)^{n-l}\sum_{k=0}^{l}(-1)^{k}\frac{l!}{k!(l-k)!}\frac{n!}{(n-k)!}A^{k}(C-\lambda A)^{(n-k)}\left(B-\frac{\partial}{\partial\lambda}\right)^{l-k}$$

$$=\left(B+\frac{\partial}{\partial\lambda}\right)^{n-(l+1)}\left[\sum_{k=0}^{l}(-1)^{k}\frac{l!}{k!(l-k)!}\frac{n!}{(n-k)!}A^{k}(C-\lambda A)^{n-k}\left(B-\frac{\partial}{\partial\lambda}\right)^{l+1-k}\right.$$

$$\left.+\sum_{\substack{k=0\\ k\to k-1}}^{l}(-1)^{k+1}\frac{l!}{k!(l-k)!}\frac{n!}{(n-k-1)!}A^{k+1}(C-\lambda A)^{n-k-1}\left(B-\frac{\partial}{\partial\lambda}\right)^{l-k}\right]$$

$$\cancel{=\left(B+\frac{\partial}{\partial\lambda}\right)^{n-(l+1)}\left[\sum_{k=0}^{l+1}(-1)^{k}\frac{l!}{k!(l-k)!}\right.}$$

$$\frac{l!}{k!(l-k)!}+\frac{l!}{(k-1)!(l+1-k)!}=\frac{(l+1)!}{k!(l+1-k)!}$$

$$\binom{l}{k}+\binom{l}{k-1}=\binom{l+1}{k}$$

Aufgabe 2:

$$\frac{\partial}{\partial\alpha}|\alpha\rangle\langle\alpha| = \frac{\partial}{\partial\alpha}\left(e^{-|\alpha|^2 \nearrow \alpha\alpha^*} e^{\alpha a^\dagger}|0\rangle\langle 0| e^{\alpha^* a}\right)$$

$$= (a^\dagger - \alpha^*)\,(|\alpha\rangle\langle\alpha|$$

$$\frac{\partial}{\partial\alpha^*}|\alpha\rangle\langle\alpha| = \frac{\partial}{\partial\alpha^*}\left(e^{-|\alpha|^2} e^{\alpha a^\dagger}\;|0\rangle\langle 0| e^{\alpha^* a}\right)$$

$$= |\alpha\rangle\langle\alpha|\,(a-\alpha)$$

Aufgabe 3

$$\dot{\rho} = -i\omega_0\left(a^\dagger a\rho - \rho a^\dagger a\right)$$

$$\int d^2\alpha \frac{\partial}{\partial t} P(\alpha,t)\;|\alpha\rangle\langle\alpha| =$$

$$\int d^2\alpha\;P(\alpha,t)\left(\left[-i\omega_0\left(a^\dagger a|\alpha\rangle\langle\alpha| - |\alpha\rangle\langle\alpha|a^\dagger a\right]\right)\right.$$

$$a^\dagger a\;|\alpha\rangle\langle\alpha| = \alpha\,a^\dagger|\alpha\rangle\langle\alpha|$$

$$= \alpha\left(\frac{\partial}{\partial\alpha} + \alpha^*\right)|\alpha\rangle\langle\alpha|$$

$$|\alpha\rangle\langle\alpha|a^\dagger a = \alpha^*|\alpha\rangle\langle\alpha|a \overset{\alpha}{\Rightarrow}$$

$$= \alpha^*\left(\frac{\partial}{\partial\alpha^*} + \alpha\right)|\alpha\rangle\langle\alpha|$$

Fortsetzung Aufg 3:

$$\int d^2\alpha\, |\alpha\rangle\langle\alpha| \frac{\partial}{\partial t} P(\alpha,t)$$

$$= \cancel{\int d^2\alpha\, P(\alpha,t)\left[-\left(\frac{\gamma}{2}+i\omega_0\right)\alpha\,\partial\right.}$$

$$= \int d^2\alpha\; P(\alpha,t)\; i\omega_0\left[-\alpha\frac{\partial}{\partial\alpha}+\alpha^*\frac{\partial}{\partial\alpha^*}\right]|\alpha\rangle\langle\alpha|$$

$$\int f g'\, dx = [f g] - \int g f'\, dx$$

$$\int d^2\alpha\, |\alpha\rangle\langle\alpha| \frac{\partial}{\partial t} P(\alpha,t)$$

$$= \int d^2\alpha\; \cancel{P(\alpha,t)}\; |\alpha\rangle\langle\alpha|$$

$$i\omega_0\left[\frac{\partial}{\partial\alpha}\alpha - \frac{\partial}{\partial\alpha^*}\alpha^*\right]P(\alpha,t)$$

$$\frac{\partial}{\partial t}P(\alpha,t) = i\omega_0\left[\frac{\partial}{\partial\alpha}\alpha - \frac{\partial}{\partial\alpha^*}\alpha^*\right]P(\alpha,t)$$