Aufgabe ~~4~~ 1

$$G^{(2)}(r_1, r_2, t_1, t_2) = \epsilon_k^4 \langle (a_k^+ e^{-ikr_1} + a_{k'}^+ e^{-ik'r_1}) (a_k^+ e^{-ikr_2} + a_{k'}^+ e^{-ik'r_2}) (a_k e^{ikr_2} + a_{k'} e^{ik'r_2}) (a_k e^{ikr_1} + a_{k'} e^{ik'r_1}) \rangle$$

$$= \epsilon_k^4 \langle a_k^+ a_k^+ a_k a_k + a_{k'}^+ a_{k'}^+ a_{k'} a_{k'} + a_k^+ a_{k'}^+ a_k a_{k'} e^{i(k-k')(r_1-r_2)} + a_k^+ a_{k'}^+ a_{k'} a_k \cancel{e^{ik}} + a_{k'}^+ a_k^+ a_k a_{k'} + a_{k'}^+ a_k^+ a_{k'} a_k e^{i(k-k')(r_1-r_2)} \rangle$$

$$= \epsilon_k^4 \langle - a_k^+ a_k a_k^+ a_k + a_k^+ a_k - a_{k'}^+ a_{k'} a_{k'}^+ a_{k'} + a_{k'}^+ a_{k'} - a_k^+ a_k a_{k'}^+ a_{k'} e^{-i(k-k')(r_1-r_2)} + a_k^+ a_k a_{k'}^+ a_{k'} + a_{k'}^+ a_{k'} a_k^+ a_k - a_{k'}^+ a_{k'} a_k^+ a_k e^{i(k-k')(r_1-r_2)} \rangle$$

$$= \epsilon_k^4 \cancel{\langle} ( -1 + 1 - 1 + 1 - e^{-i(k-k')(r_1-r_2)} + 1 + 1 - e^{+i(k-k')(r_1-r_2)} )$$

$$= 2\epsilon_k^4 (1 - \cos((k-k')(r_1-r_2)))$$

Aufgabe R

$$\vec{E}(\vec{r}) = i \sum_{\vec{k},\mu} \sqrt{\frac{\hbar\,\omega_k}{2V\epsilon_0}} \left( \vec{e}^{(\mu)} a^{(\mu)}_{\vec{k}} e^{i\vec{k}\cdot\vec{r}} - \vec{e}^{(\mu)*} a^{+(\mu)}_{\vec{k}} e^{-i\vec{k}\vec{r}} \right)$$

$$\vec{B}(\vec{r}) = i \sum_{\vec{k},\mu} \sqrt{\frac{\hbar}{2\omega_k V\epsilon_0}} \Big( (\vec{k} \times \vec{e}^{(\mu)}) a^{(\mu)}_{\vec{k}} e^{i\vec{k}\vec{r}} - (\vec{k} \times \vec{e}^{(\mu)}) a^{+(\mu)}_{\vec{k}} e^{-i\vec{k}\vec{r}} \Big)$$

$a_k \to a_k e^{-i\omega_k t} \quad a_k^+ \to a_k^+ e^{+i\omega_k t}$

$$\vec{P} = \int d^3\vec{r}\ \vec{E} \times \vec{B} \qquad \int d^3\vec{r}\, e^{ikr} e^{-ik'r} = V\delta_{kk'}$$

$$\vec{e}^{(\mu)} \times (\vec{k} \times \vec{e}^{(\mu')}) = \vec{k}\,(\vec{e}^{(\mu)}\cdot\vec{e}^{(\mu')}) + e^{(\mu')} (\underbrace{\vec{e}^{(\mu)} \cdot \vec{k}}_{0})$$

0 per definition von $\vec{e}^{(\mu)}$

$$= \vec{k}\,\delta_{\mu\mu'}$$

$$\Rightarrow \quad \epsilon_0 \cdot \sqrt{\frac{\hbar\,\omega_k}{2V\epsilon_0}} \sqrt{\frac{\hbar}{2\omega_k V\epsilon_0}} = \frac{\hbar}{2V}$$

$$\Rightarrow \quad \vec{P} = \sum_{\vec{k},(\mu)} \hbar\vec{k} \left( a^{+(\mu)}_{\vec{k}} a^{(\mu)}_{\vec{k}} + a^{(\mu)}_{\vec{k}} a^{+(\mu)}_{\vec{k}} \right)$$

$$= \sum_{\vec{k},(\mu)} \hbar\vec{k} \left( a^{+(\mu)}_{\vec{k}} a^{(\mu)}_{\vec{k}} + \frac{1}{2} \right)$$