

Aufgabe 1

$$G^{(2)}(r_1, r_2, t_1, t_2) = \epsilon_n^4 \left\langle \begin{aligned} &(a_{k'}^\dagger e^{-ik'r_1} + a_{k'} e^{-ik'r_1}) \\ &(a_k^\dagger e^{-ikr_2} + a_k e^{-ikr_2}) \\ &(a_k e^{ikr_2} + a_{k'} e^{ik'r_2}) \\ &(a_n e^{ikr_1} + a_{n'} e^{ik'r_1}) \end{aligned} \right\rangle$$

$$= \epsilon_n^4 \left\langle a_{k'}^\dagger a_n^\dagger a_k a_n + a_{k'}^\dagger a_{k'}^\dagger a_{k'} a_{k'} \right.$$

$$+ a_k^\dagger a_{k'}^\dagger a_k a_{k'} e^{i(k-k')(r_1-r_2)} \left. \right.$$

$$+ a_n^\dagger a_{k'}^\dagger a_{k'} a_n + a_{k'}^\dagger a_k^\dagger a_k a_{k'} \left. \right.$$

$$+ a_{k'}^\dagger a_n^\dagger a_{k'} a_n e^{i(k-k')(r_1-r_2)} \left. \right\rangle$$

$$= \epsilon_n^4 \left\langle -a_n^\dagger a_n a_{k'}^\dagger a_{k'} + a_{k'}^\dagger a_{k'} \right.$$

$$- a_{k'}^\dagger a_{k'} a_n^\dagger a_n + a_{k'}^\dagger a_{k'} \left. \right.$$

$$- a_n^\dagger a_n a_{k'}^\dagger a_{k'} e^{-i(k-k')(r_1-r_2)} \left. \right.$$

$$+ a_{k'}^\dagger a_n a_{k'}^\dagger a_{k'} + a_{k'}^\dagger a_{k'} a_n^\dagger a_n \left. \right.$$

$$- a_{k'}^\dagger a_{k'} a_n^\dagger a_n e^{i(k-k')(r_1-r_2)} \left. \right\rangle$$

$$= \epsilon_n^4 \left\langle \begin{aligned} &-1 + 1 - 1 + 1 - e^{-i(k-k')(r_1-r_2)} \\ &+ 1 + 1 - e^{+i(k-k')(r_1-r_2)} \end{aligned} \right.$$

$$\left. \right\rangle$$

$$= 2\epsilon_n^4 (1 - \cos((k-k')(r_1-r_2)))$$

Aufgabe 2

$$\vec{E}(\vec{r}) = i \sum_{\vec{k}, \mu} \sqrt{\frac{\hbar \omega_{\vec{k}}}{2V \epsilon_0}} \left(\vec{e}^{(\mu)} a_{\vec{k}}^{(\mu)} e^{i\vec{k} \cdot \vec{r}} - \vec{e}^{(\mu)*} a_{\vec{k}}^{+(\mu)} e^{-i\vec{k} \cdot \vec{r}} \right)$$

$$\vec{B}(\vec{r}) = i \sum_{\vec{k}, \mu} \sqrt{\frac{\hbar}{2\omega_{\vec{k}} V \epsilon_0}} \left((\vec{k} \times \vec{e}^{(\mu)}) a_{\vec{k}}^{(\mu)} e^{i\vec{k} \cdot \vec{r}} - (\vec{k} \times \vec{e}^{(\mu)}) a_{\vec{k}}^{+(\mu)} e^{-i\vec{k} \cdot \vec{r}} \right)$$

$$a_{\vec{k}} \rightarrow a_{\vec{k}} e^{-i\omega_{\vec{k}} t} \quad a_{\vec{k}}^+ \rightarrow a_{\vec{k}}^+ e^{+i\omega_{\vec{k}} t}$$

$$\vec{p} = \int d^3\vec{r} \vec{E} \times \vec{B} \quad \int d^3\vec{r} e^{i\vec{k} \cdot \vec{r}} e^{-i\vec{k}' \cdot \vec{r}} = V \delta_{\vec{k}, \vec{k}'}$$

$$\vec{e}^{(\mu)} \times (\vec{k} \times \vec{e}^{(\mu')}) = \vec{k} (\vec{e}^{(\mu)} \cdot \vec{e}^{(\mu')})$$

$$+ \vec{e}^{(\mu')} (\underbrace{\vec{e}^{(\mu)} \cdot \vec{k}}_0)$$

0 per definition von $\vec{e}^{(\mu)}$

$$= \vec{k} \delta_{\mu, \mu'}$$

$$\Rightarrow G_0 \cdot \sqrt{\frac{\hbar \omega_{\vec{k}}}{2V \epsilon_0}} \sqrt{\frac{\hbar}{2\omega_{\vec{k}} V \epsilon_0}} = \frac{\hbar}{2V}$$

$$\Rightarrow \vec{p} = \sum_{\vec{k}, \mu} \hbar \vec{k} \left(a_{\vec{k}}^{+(\mu)} a_{\vec{k}}^{(\mu)} + a_{\vec{k}}^{(\mu)} a_{\vec{k}}^{+(\mu)} \right)$$

$$= \sum_{\vec{k}, \mu} \hbar \vec{k} \left(a_{\vec{k}}^{+(\mu)} a_{\vec{k}}^{(\mu)} + \frac{1}{2} \right)$$