Aufgabe 1: Blatt 8

$n = 10^{11}\ \mathrm{cm}^{-2}$

$\mu = 10^{7}\ \frac{\mathrm{cm}^2}{\mathrm{Vs}}$

Def.: Driftgeschw. $v_D = \mu E$

Drude Bild: $\underbrace{\frac{1}{\tau}}_{\text{Zeit zwischen Stössen.}} m v_d = \underbrace{eE}_{\text{Kraft}}$

$\Rightarrow \quad v_d = \frac{eE\tau}{m} = \mu E \quad \Rightarrow \quad \tau = \frac{\mu m}{e}$

$\Rightarrow \quad L = \tau v_F = \frac{\mu m}{e} v_F = \frac{\mu}{e} p_F$

2D: $\Rightarrow \quad \cancel{2\pi} \quad 2 \frac{\pi p_F^2}{(2\pi\hbar)^2} = n$

$\Rightarrow \quad p_F = \hbar \sqrt{n} \sqrt{2\pi}$

$\Rightarrow \quad L = \frac{\mu}{e} \hbar \sqrt{n} \sqrt{2\pi} = 0.5\ \mu m$

Aufgabe 2:

Dresselhaus Spin Orbit Kopplung:

$$H = \frac{p^2}{2m} + \beta(\sigma_y p_y - \sigma_x p_x)$$

~~Damit~~: Ansatz: $\psi = \chi\, e^{i(p_x x + p_y y)/\hbar}$

$\cancel{\chi} \Rightarrow \bar{H}\chi = E\chi$

$$\bar{H} = \beta(\sigma_y p_y - \sigma_x p_x) = -\begin{pmatrix} 0 & p_x + ip_y \\ p_x - ip_y & 0 \end{pmatrix}\beta$$

$$= \beta\begin{pmatrix} 0 & z \\ z^* & 0 \end{pmatrix} \qquad z = p_x + ip_y$$

$|\bar{H}| = -\beta^2 |z|^2$ $\cancel{|\bar{H}| \, |\bar{H}| \, |\bar{H}|}$

$$|\bar{H} - \lambda \mathbb{1}| = \lambda^2 - |z|^2 \beta^2 \Rightarrow E = \pm |z| \beta$$

$$= \pm \sqrt{p_x^2 + p_y^2}\, \beta$$

bzw: $E = \frac{p^2}{2m} \pm \sqrt{p_x^2 + p_y^2}\, \beta$

$$\left.\begin{aligned} \psi_+ &= \begin{pmatrix} p_x + ip_y \\ \sqrt{p_x^2 + p_y^2} \end{pmatrix} e^{i(p_x x + p_y y)/\hbar} \\ \psi_- &= \begin{pmatrix} -(p_x + ip_y) \\ \sqrt{p_x^2 + p_y^2} \end{pmatrix} e^{i(p_x x + \cancel{p_y} y)/\hbar} \end{aligned}\right\} \text{unnormiert}$$

mit $p = \sqrt{p_x^2 + p_y^2}$ und $p_x + ip_y = e^{i\varphi} p$

$$\Rightarrow \psi_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\varphi/2} \\ e^{-i\varphi/2} \end{pmatrix} e^{i \cdots} \qquad \psi_- = \frac{1}{\sqrt{2}}\begin{pmatrix} -e^{i\varphi/2} \\ e^{-i\varphi/2} \end{pmatrix} e^{i \cdots}$$

Aufgabe 3:

Genau wie Aufg. 2:

$\bar{H}\chi = E\chi \Rightarrow \bar{H} = \begin{pmatrix} -a & b \\ b^* & a \end{pmatrix}$

$a = \mu_0 B \qquad b = \alpha(p_y + i p_x)$

$$|\bar{H} - \lambda \mathbb{1}| = +(\lambda + a)(\lambda - a) - |b|^2 \overset{!}{=} 0$$

$$= \lambda^2 - a^2 - |b|^2$$

$$\Rightarrow \lambda = \pm\sqrt{a^2 + |b|^2}$$

Es ist sinnvoll $\bar{H}$ umzuschreiben um

$$\bar{H} = c\begin{pmatrix} -\cos\theta & \sin\theta\, e^{i\varphi} \\ \sin\theta\, e^{-i\varphi} & \cos\theta \end{pmatrix}$$

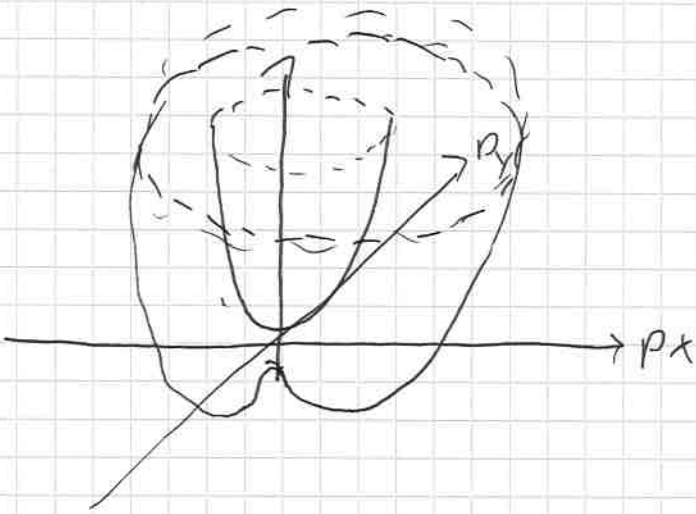
$$c = \sqrt{a^2 + |b|^2}$$

$$\Rightarrow \psi_+ = \begin{pmatrix} e^{i\varphi/2} \sin(\theta/2) \\ e^{-i\varphi/2} \cos(\theta/2) \end{pmatrix}$$

$$\psi_- = \begin{pmatrix} -e^{i\varphi/2} \cos(\theta/2) \\ e^{i\varphi/2} \sin(\theta/2) \end{pmatrix}$$

$$E = \frac{p^2}{2m} \pm \sqrt{(\mu_0 B)^2 + \alpha^2 p_y^2 + \alpha^2 p_x^2}$$

Aufgabe 3:



$$\left.\begin{array}{l} E_+ = \dfrac{p^2}{2m} + \sqrt{(\mu_0 B)^2 + \alpha^2 p^2} \xrightarrow{(B=0)} \dfrac{p^2}{2m} + p \\ E_- = \dfrac{p^2}{2m} - \cancel{+} \sqrt{(\mu_0 B)^2 + \alpha^2 p^2} \xrightarrow{(B=0)} \dfrac{p^2}{2m} - p \end{array}\right\} \begin{array}{l} = 0 \\ \text{for } p = 0 \end{array}$$

Aufgabe 4:

$$S = \begin{pmatrix} r & t & t \\ t & r & t \\ t & t & r \end{pmatrix}$$

Unitarität gibt uns 2 Bedingungen:

$$|r|^2 + 2|t|^2 = 1 \qquad a$$
$$t r^* + r t^* + |t|^2 = 0 \qquad b$$

wir wählen r ist reell und $t = -t_0 e^{i\phi}$

$$\Rightarrow t_0 \left[2r\cos(\phi) - t_0\right] = 0 \qquad b$$

$$\Rightarrow t_0 = 2r\cos\phi$$

mit a

$$r^2 + 8 r^2 \cos^2\phi = 1$$

$$\Rightarrow r = \frac{1}{\sqrt{1+8\cos^2\phi}}$$

$$t = -\frac{2\cos(\phi)}{\sqrt{1+8\cos^2\phi}} e^{i\phi}$$

$$\phi = 0 \Rightarrow r = \frac{1}{\sqrt{1+8}} = \frac{1}{3}$$

$$\phi = \pi \Rightarrow r = 1$$