①

$$\vec{j}(\vec{r}) = -e \int d\epsilon \, N(\epsilon) \int \frac{d\hat{p}}{4\pi} \, \vec{v}(\hat{p}) \, f(\vec{r}, \hat{p}, \epsilon)$$

$$f(\vec{r}, \hat{p}, \epsilon) = f_0(\vec{r}, \epsilon) + \vec{\delta f}(\vec{r}, \epsilon) \cdot \hat{p}$$

$$\int \frac{d\hat{p}}{4\pi} \, \vec{v}(\hat{p}) \, f_0(\vec{r}, \epsilon) = 0$$

$$\int \frac{d\hat{p}}{4\pi} \, \hat{v}(\hat{p}) \, \vec{\delta f} \, \hat{p} = \frac{1}{3} v \, \vec{\delta f}$$

von Blatt 6: $v \nabla f_0 = -\frac{1}{\tau} \vec{\delta f}$

$$\Rightarrow \vec{\delta f} = -\tau v \nabla f_0$$

$$\Rightarrow \int \frac{d\hat{p}}{4\pi} \, v \, \vec{\delta f} \, \hat{p} = -\underbrace{\frac{1}{2} v^2 \tau}_{D} \nabla f_0$$

$$\Rightarrow \vec{j} = e \int d\epsilon \, N(\epsilon) \, \cancel{\int \frac{d\hat{p}}{4\pi}} \, D \, \nabla f_0$$

(2)

allg. Lösung von $D \partial_x^2 f(x) = 0$

$$f(x) = a x + b$$

$$f(0) = b = f_L \qquad f(L) = aL + f_L = f_R$$

$$\Rightarrow \cancel{a = f_L \; f_R}$$

$$a = (f_R - f_L)/L$$

$$\Rightarrow f(x) = \frac{f_R - f_L}{L} x + f_L$$

$$= f_R \frac{x}{L} + f_L \left(1 - \frac{x}{L}\right)$$

$$\partial_x f(x) = \frac{1}{L}\left(f_R - f_L\right)$$

$$\Rightarrow \quad j = e \nu D \int d\epsilon \, \partial_x f(x)$$

$$= e \nu D \int d\epsilon \, \frac{1}{L}\left(f_R - f_L\right)$$

$$= \frac{e^2 \nu D}{L} V = \frac{\sigma}{L} V$$

$$\sigma = e^2 \nu D$$

(3)

Ansatz

$$f_1(x) = \left(1 + \frac{x}{L}\right) f_0 - \frac{x}{L} f_L \qquad -L < x < 0$$

$$f_2(x) = \left(1 - \frac{x}{L}\right) f_0 + \frac{x}{L} f_R \qquad 0 < x < L$$

$$\cancel{\Rightarrow j_L = \frac{e D \mathbb{N}}{L} \left(\mu_0 - \mu_0\right)}$$

$$j_L = e \mathbb{N}_L D_L \int dE \, \partial_x f_1$$

$$= e \mathbb{N}_L D_L \int dE \left(f_0 - f_L\right)$$

$$= e \mathbb{N}_L D_L \left(\mu_0 - \mu_L\right) \overset{!}{=} j_R$$

$$= e \mathbb{N}_R D_R \left(\mu_R - \mu_0\right)$$

$$\left(e \mathbb{N}_L D_L + e \mathbb{N}_R D_R\right) \mu_0 = e \mathbb{N}_L D_L \mu_L + e \mathbb{N}_R D_R \mu_R$$

$$\Rightarrow \quad \mu_0 = \frac{\mathbb{N}_L D_L \mu_L + \mathbb{N}_R D_R \mu_R}{\mathbb{N}_L D_L + \mathbb{N}_R D_R}$$

4

a) In einer Dimension:

$$\cancel{\partial_x^2 (\mu')_\sigma} \partial_x^2 f_{\sigma} = \frac{f_{\bar\sigma} - f_\sigma}{2\tau_\sigma}$$

$$f_\sigma = \frac{1}{e^{\beta(\epsilon - \mu_\sigma)} + 1} \qquad \mu_\sigma = \mu_\sigma(x) \qquad \tilde\epsilon = \epsilon - \mu_\sigma$$

$$\partial_x^2 f_\sigma = \frac{2\beta^2 e^{2\beta\tilde\epsilon} \mu_\sigma'^2}{(1 + e^{\beta\tilde\epsilon})^3} - \frac{\beta^2 e^{\beta\tilde\epsilon} \mu_\sigma'^2}{(1 + e^{\beta\tilde\epsilon})^2} + \frac{\beta e^{\beta\tilde\epsilon} \mu_\sigma''}{(1 + e^{\beta\tilde\epsilon})^2}$$

$$= \frac{2\beta^2 e^{\frac{1}{2}\beta\tilde\epsilon} \mu_\sigma'^2}{(e^{-\frac{\beta}{2}\tilde\epsilon} + e^{\frac{\beta}{2}\tilde\epsilon})^3} - \frac{\beta^2 \mu_\sigma'^2}{(e^{-\frac{\beta}{2}\tilde\epsilon} + e^{\frac{\beta}{2}\tilde\epsilon})^2} + \frac{\beta e^{\beta\tilde\epsilon} \mu_\sigma''}{(1 + e^{\beta\tilde\epsilon})^2}$$

$$= \frac{2e^{\frac{1}{2}\beta\tilde\epsilon} - e^{-\frac{\beta}{2}\tilde\epsilon} - e^{\frac{\beta}{2}\tilde\epsilon}}{8\cosh[\beta\tilde\epsilon/2]} \beta^2 \mu_\sigma'^2 + \dots$$

$$= \underbrace{\frac{\sinh(\beta\tilde\epsilon/2)}{4\cosh(\beta\tilde\epsilon/2)} \beta^2 \mu_\sigma'^2}_{\text{assymetrisch in } \tilde\epsilon} + \frac{\beta\, \mu_\sigma''}{4\cosh(\beta\tilde\epsilon/2)}$$

$$\Rightarrow \int d\tilde\epsilon\, \partial_x^2 f_\sigma = \partial_x^2 \mu_\sigma \int_{-\infty}^{\infty} d\tilde\epsilon\, \frac{\beta}{4\cosh(\beta\tilde\epsilon/2)}$$

$$\int_{-\infty}^{\infty} d\tilde{\epsilon} \; \frac{\beta}{4\cosh(\beta\tilde{\epsilon}/2)^2} = \int_{-\infty}^{\infty} dx \; \frac{1}{\cosh(x/2)^2}$$

$$= \int_{-\infty}^{\infty} dx \; \frac{2e^{\cancel{x/2}}}{(1+e^{x})^2} = \int_0^{\infty} dy \; \frac{1}{y} \; \frac{2y}{(1+y)^2}$$

$$y = e^x \qquad \frac{dy}{dx} = y$$

$$= \int_0^{\infty} dy \; \frac{2}{(1+y)^2} = \left[-\frac{1}{1+y} \right]_0^{\infty} = 1$$

$$\Rightarrow D_\sigma \partial_x^2 \mu_\sigma = \frac{\mu_\sigma^{*} - \mu_{\bar{\sigma}}}{2 J^\sigma}$$

$$\Rightarrow \partial_x^2 \mu_\sigma = \frac{\mu_\sigma^{*} - \mu_{\bar{\sigma}}}{2\, l_\sigma^2}$$

$$l_\sigma = \sqrt{D_\sigma J^\sigma}$$

~~b)~~

$$\partial_x f_\sigma = \cancel{\frac{\beta}{(1+e^{\beta(\epsilon-\mu)})}} \frac{\beta}{4\cosh(\beta\tilde{\epsilon}/2)} \mu_\sigma'$$

$$\Rightarrow j = e N_\sigma D_\sigma \int d\epsilon \, \partial_x f_\sigma$$

$$= e N_\sigma D_\sigma \partial_x \mu_\sigma \int d\tilde{\epsilon} \frac{\beta}{4\cosh(\beta\tilde{\epsilon}/2)}$$

$$= e N_\sigma D_\sigma \partial_x \mu_\sigma$$

b) Gegeben:

$$\mu_{\uparrow/\downarrow} = \frac{c_\downarrow^2 (c_1 + c_2 x) + c_\uparrow^2 (c_3 + c_4 x)}{c_{tot}^2}$$

$$\pm \frac{c_{\downarrow\uparrow}^2}{c_{tot}^2} \left[(c_3 - c_1) \cosh(x/l) + l\,(c_4 - c_2) \sinh(x/l) \right]$$

$$\mu_{\uparrow} - \mu_\downarrow = \underbrace{\frac{2(c_\downarrow^2 + c_\uparrow^2)}{c_{tot}^2}}_{=1} \left[(c_3 - c_1) \cosh(x/l) + l\,(c_4 - c_2) \sinh(x/l) \right]$$

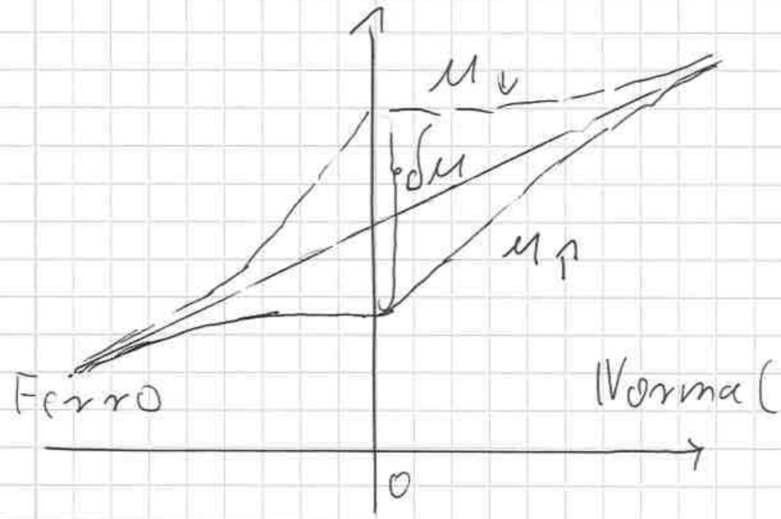
$$\partial_x^2 \mu_{\uparrow/\downarrow} = \pm \frac{1}{l^2} \frac{c_{\downarrow\uparrow}^2}{c_{tot}^2} \left((c_3 - c_1) \cosh(x/l) + l\,(c_4 - c_2) \sinh(x/l) \right)$$

$$= \pm \frac{1}{2 l_{\uparrow\downarrow}} \left((c_3 - c_1) \cosh(x/l) + l\,(c_4 - c_2) \sinh(x/l) \right)$$

$$\Rightarrow \quad \partial_x^2 \mu_{\uparrow/\downarrow} = \frac{\mu_\uparrow - \mu_\downarrow}{2 l_0}$$

c)

Skizze für $\sigma_F = \sigma_N$:



$$C_\uparrow^2 \partial_x \mu_\uparrow + C_\downarrow^2 \partial_x \mu_\downarrow$$

$$= (C_\uparrow^2 + C_\downarrow^2)\frac{ej}{2\sigma_F} + \underbrace{\frac{C_\uparrow^2 + C_\downarrow^2}{l} e^{x/l} \delta\mu \frac{C_\downarrow^2 - C_\uparrow^2}{2C_{tot}^2} + \frac{C_\uparrow^2 - C_\downarrow^2}{l} \frac{\delta\mu}{2} e^{x/l}}_{0}$$

$$= \frac{C_\uparrow^2 + C_\downarrow^2}{2\sigma_F} ej = \frac{C_\uparrow^2 + C_\downarrow^2}{e^2(N_\uparrow D_\uparrow + N_\downarrow D_\downarrow)} ej = \frac{j}{e\sigma}$$