Properties of thermal radiation in photonic crystals

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Abstract
We analyze the properties of the thermal radiation in photonic crystals and show that the spectral energy density, the spectral intensity, and the spectral hemispherical power are only limited by the total number of photonic states available. Due to the possible presence of photonic band gaps and the associated spectral redistribution of photonic states, it is possible to have spectral regions over which the thermal radiation generation and propagation are completely inhibited or strongly enhanced relative to the corresponding free space or homogeneous dielectric medium blackbody limits. Using a differential geometry approach, we also investigate the directional properties of the thermal radiation propagation in photonic crystals and analyze the thermal photon focusing effects that take place in these systems.

Keywords: photonic crystals, photonic band gap materials, thermal radiation, thermal emissivity

(Some figures in this article are in colour only in the electronic version)

1. Introduction
Photonic crystals, and in particular photonic band gap (PBG) materials, constitute a novel class of dielectric materials, in which the basic electromagnetic interaction is controllably altered over certain frequencies and length scales \([1–6]\). The ability to tailor the optical properties in a prescribed manner and the specific symmetry properties of the electromagnetic field inside a PBG material are the fundamental instruments for designing materials and devices that control the emission and absorption properties of light \([7–13]\).

However, there is more to photonic crystals than a PBG. For instance, the modification of spontaneous emission from atoms or semiconductors embedded in photonic crystals \([7]\) determines important alterations of thermal radiative processes \([14]\). Thermal radiation can be regarded as spontaneous emission that is thermally driven and is in thermal equilibrium with its material surroundings. This represents the key ingredient for achieving significant modifications of Planck’s blackbody radiation spectrum through the optimization of the coupling between the multi-mode radiation field of a photonic crystal and a collection of emitters \([15, 16]\).

From the definition of a blackbody object\(^5\) it follows that, for any specific frequency and direction, the thermal flux of the propagating radiation emitted by any material object cannot exceed the corresponding flux emitted by a blackbody at the same temperature and placed in identical surroundings. This does not imply anything regarding the nature of either the blackbody object or the photonic environment in which the blackbody is placed. Rather, the blackbody object is defined in a manner that is independent of the surrounding photonic environment. The only assumptions made are that all systems are assumed to be in thermal equilibrium and that we are considering only the flux of thermal radiation propagating away from the radiating/absorbing object. Evanescent fields,

\(^5\) A blackbody is defined as a material object that for all directions and frequencies absorbs all of the thermal radiation incident upon it. Since at thermal equilibrium the thermal flux radiated by the blackbody is equal to the thermal flux absorbed, it follows that, at thermal equilibrium, the blackbody object emits the maximum possible flux of thermal radiation.
for which the thermal radiation is localized inside or at the surface of the object, do not contribute to the thermal flux of the propagating radiation. In fact, it has long been recognized that evanescent fields can generate ‘super-Planckian’ thermal radiation flux as opposed to the Planckian transfer in the far-field regime [17, 18]. The surrounding photonic environment is an important factor in determining the properties of the emitted and absorbed thermal radiation. For example, the radiation emitted by a blackbody object in free space turns out to be given by the well known Planck law, and only in this case can a blackbody be defined as a material object whose emission and absorption of radiation follow Planck’s law. In general, the photonic reservoir associated with the surrounding medium may support only a prescribed number of photon modes for a given frequency and propagating direction. In other words, while a blackbody object placed in free space emits/absorbs at thermal equilibrium into/from the surrounding space a thermal flux that is prescribed by Planck’s law, the very same blackbody object placed in an ‘infinite’ photonic crystal emits/absorbs thermal flux, which, depending on frequency and propagating direction, may exceed or fall below the thermal flux emitted in free space.

Despite the recent surge in attention in the thermal radiation properties of photonic crystals, and the important advances made both in its experimental [14] and the theoretical characterization [20], most studies [14, 20, 21] invoke the frequency dependence of the photonic density of states (DOS) for the interpretation of their findings. In this study, we further explore the theoretical framework [22] and show that the central quantity that describes thermal radiation transport properties such as intensity, emissive power, and absorptance is in fact the area of the iso-frequency surfaces rather than the photonic DOS. The latter is proportional to the infinitesimal volume around these surfaces and essentially describes the energy density of the system and not the energy current density. We illustrate our results through an investigation of a two-dimensional model photonic crystal.

2. Spectral energy density of the thermal radiation

To begin, we analyze the thermal emission and absorption of radiation from elementary emitter/absorbers that are placed inside non-magnetic media with translational invariant static properties (translational invariant dielectric permittivity \(\epsilon(r)\) function). This includes homogeneous and isotropic dielectric media (where the system is invariant under infinitesimal translation operations), and photonic crystals (where the system is invariant under a discrete set of translation operations associated with a Bravais lattice).

The quantization of the electromagnetic field in photonic crystals follows the conventional path of the free-space canonical field quantization scheme (see [23–26]), and the Hamiltonian is given by

\[
\hat{H} = \frac{1}{V} \int_V d\mathbf{r} \hat{H}(\mathbf{r}, \mathbf{t}) = \frac{1}{V} \sum_k \hbar \omega_k \left\{ \hat{a}_{k\mathbf{r}} \hat{\mathbf{a}}_{k\mathbf{r}}^+ + \frac{1}{2} \right\}.
\]  

(2.1)

Here, \(\hat{a}_{k\mathbf{r}}, \hat{\mathbf{a}}_{k\mathbf{r}}^+\) are the usual bosonic creation and annihilation operators of the elementary excitations (photons) of the electromagnetic field modes (either plane waves or Bloch functions)

\[
[\hat{a}_{k\mathbf{r}}, \hat{a}_{k'\mathbf{r}'}^+] = \left[ \hat{a}_{k\mathbf{r}}^+, \hat{a}_{k'\mathbf{r}'} \right] = 0, \quad (2.2a)
\]

\[
[\hat{a}_{k\mathbf{r}}, \hat{\mathbf{a}}_{k'\mathbf{r}'}^+] = \delta_{kk'} \delta_{\mathbf{r} \mathbf{r}'}.
\]  

(2.2b)

In the context of the formalism presented above, we consider a gas of photons in a weakly absorbing dielectric medium (index of refraction \(n(\mathbf{r}) = \sqrt{\epsilon(\mathbf{r})}\)) with an unspecified dispersion relation \(\omega_k\), that is in thermal equilibrium at temperature \(T\) and is contained in a volume \(V\). The density operator for the thermal radiation reservoir is

\[
\hat{R}_T = \prod_k e^{\hat{h} \omega_k \hat{a}_{k\mathbf{r}} / \hbar \omega_k T} [1 - e^{\hat{h} \omega_k \hat{a}_{k\mathbf{r}} / \hbar \omega_k T}].
\]  

(2.3)

Of more practical interest is to spectrally decompose the local and average thermal energy densities. Consequently, we introduce the average spectral energy density \(U(\omega, T)\) through

\[
\langle H(\omega, T) \rangle = \int_0^\infty d\omega U(\omega, T),
\]  

(2.4)

where \(\langle \rangle\) is used to denote averaging over the thermal reservoir. The average thermal energy density \(\langle H(\omega, T) \rangle\) can be evaluated using

\[
\langle H(\omega, T) \rangle = \frac{1}{V} \sum_k \hbar \omega_k n_k(T) \int_0^\infty d\omega \delta(\omega - \omega_k)
\]

\[
= \int_0^\infty d\omega \hbar \omega n_\text{int}(\omega, T) \rho(\omega).
\]  

(2.5)

Here, we have introduced the average number of photons with frequency \(\omega\)

\[
n(\omega, T) = [e^{\hbar \omega / k_B T} - 1]^{-1},
\]  

(2.6)

and the photonic DOS, i.e. the number of photon modes at frequency \(\omega\) per unit volume

\[
\rho(\omega) = \frac{1}{V} \sum_k \delta(\omega - \omega_k).
\]  

(2.7)

From equations (2.4) and (2.5), we thus find

\[
U(\omega, T) = \hbar \omega n_\text{int}(\omega, T) \rho(\omega).
\]  

(2.8)

For future reference it is useful to consider the well known case of a weakly absorbing homogeneous dielectric medium (of index of refraction \(n = \sqrt{\epsilon}\)) explicitly. We then have an isotropic dispersion relation \(\omega_k = c / n |\mathbf{k}|\) and the photon modes are plane waves so that the mode index becomes \(\lambda \rightarrow |\mathbf{k}, \sigma|\), where \(\mathbf{k}\) and \(\sigma\) are the photon wavevector and its polarization. The corresponding three-dimensional photonic DOS is given by

\[
\rho_{\text{hd}}(\omega) = n^3 \omega^2 / \pi^2 c^3,
\]  

(2.9)

so that the ratio of the homogeneous dielectric radiative energy density with respect to the free-space case is

\[
U_{\text{hd}}(\omega, T) = n^3 U_{\lambda}(\omega, T).
\]  

(2.10)
Here, \( U_{ls}(\omega, T) \) is the free-space radiative energy density given by the well known Planck formula [27]

\[
U_{ls}(\omega, T) = \frac{\hbar \omega^3}{\pi^2 c^3} \frac{1}{e^{\hbar \omega / k_B T} - 1}.
\]

(2.11)

Clearly, the blackbody radiation in a homogeneous dielectric medium is enhanced relative to that of free space because an index of refraction \( n \neq 1 \) corresponds to an optically denser medium. Since the photonic DOS depends on the dimensionality of the system, this enhancement also depends on the dimensionality of the system. More generally, we have

\[
\frac{U_{bd,l}(\omega, T)}{U_{ls}(\omega, T)} = \frac{\rho_{bd,l}(\omega)}{\rho_{ls}(\omega)} = n^d, \quad (2.12)
\]

where \( d \) is the system dimensionality.

We now turn our attention to the case of a photon gas that is in thermal equilibrium at temperature \( T \) within a photonic-crystal environment. The corresponding modes have specific symmetry properties that are related to the particular geometrical arrangement and refractive indices of the constituent materials. As a consequence, the plane-wave expansion of homogeneous media is replaced by an expansion into this Bloch mode basis. The mode index \( \lambda \) is then replaced by the Bloch mode index \( [n, k] \), where \( n \) is the energy band index, and the \( k \)-vector is confined to the first Brillouin zone (1BZ) [26]. \( \omega_{n,k} \) represents the dispersion relation in the photonic crystal, which strongly depends on the mode index\(^\text{6}\).

For specific frequencies and propagation direction, the photonic-crystal reservoir does not support any propagating modes, while for other well defined frequencies and propagation directions, the thermal absorption/emission is favored by the availability of a large number of photonic modes. The enhancement and the suppression of the energy density in photonic crystals can thus be traced to the photonic DOS. Therefore, the average spectral energy density inside a photonic crystal is given by

\[
U_{pc}(\omega, T) = U_{ls}(\omega, T) \frac{\rho_{pc}(\omega)}{\rho_{ls}(\omega)}. \quad (2.13)
\]

Depending on the frequency, the photonic DOS in a photonic crystal can either be enhanced or suppressed relative to free space near, respectively, photonic band-edges and other van Hove singularities or over spectral ranges that correspond to complete band gaps or stop gaps [28]. As a result, the radiative energy density inside a photonic crystal may exceed the free-space radiative energy density given by Planck’s law, equation (2.11). This modification of Planck’s law is achieved without changing the properties of the absorber/emitter medium.

### 3. Flow of thermal radiation

We now turn to the analysis of the thermal radiation flow in an infinite photonic crystal and evaluate the directional

\(^6\) In a photonic crystal, for a given wavevector \( k \), different polarization states are not necessarily degenerate, since the anisotropy of the photonic crystal may lift the polarization degeneracy. Consequently, the band index \( n \) also counts the polarization of the given state.

and spectral distributions of the thermal radiation intensity and emissive power. As compared to the well known free-space case, this analysis is more tedious since each mode is characterized by its own energy transport velocity and the introduction of a spectral representation is more cumbersome.

The flow of electromagnetic energy in photonic crystals is described by the time-averaged Poynting’s vector, which is defined as the energy per unit time per unit area that is carried by the electromagnetic field. The time-averaged Poynting vector can be resolved into contributions of individual Bloch modes according to

\[
\hat{S}(r) = \text{Re}[\hat{E}(r, t)] \times \text{Re}[\hat{H}(r, t)] = \sum_{n,k} \hat{S}_{n,k}(r), \quad (3.1)
\]

where \( \hat{E}_{n,k}(r) \) and \( \hat{H}_{n,k}(r) \) are the quantized electric and magnetic fields. The total flux of the thermal electromagnetic radiation through an arbitrary surface \( \Sigma \) is thus given by

\[
\hat{\Phi}(\Sigma) = \sum_{n,k} \hat{\Phi}_{n,k}(\Sigma) = \sum_{n,k} \int_{\Sigma} d\Sigma \cdot \hat{S}_{n,k}(r), \quad (3.3)
\]

where we have accounted for the incoherent character of the thermal radiation and have neglected the interference terms.

The evaluation of the Poynting vector and the thermal radiation flux defined above requires intensive numerical computations. In the following, we adopt a somewhat simplified approach and spatially average the flow of the electromagnetic radiation over a photonic-crystal unit cell. To this end, we use the fact that in a photonic crystal the energy velocity coincides with the group velocity [25], and decompose the energy velocity as

\[
v^{(c)}_{n,k} = v^{(g)}_{n,k} \in_{n,k} \equiv \frac{\partial \omega_{n,k}}{\partial k}, \quad (3.4)
\]

where \( v^{(g)}_{n,k} \) and \( \in_{n,k} \) are the magnitude and the direction of the group velocity, respectively. Then, the spatially averaged energy current of thermal radiation for a given Bloch mode, \( J_{n,k}(T) \), reads as

\[
J_{n,k}(T) \equiv \frac{1}{V} \int_V dr \hat{S}_{n,k}(r) = i_{n,k}(T) \in_{n,k}. \quad (3.5a)
\]

Here, we have introduced

\[
i_{n,k}(T) = v^{(g)}_{n,k} u_{n,k}(T), \quad (3.5b)
\]

and

\[
 u_{n,k}(T) = \frac{\hbar \omega_{n,k}}{\rho_{\text{fs},k} c^2 k_B T} \quad (3.5c)
\]

We now proceed and introduce certain physical quantities that are usually used to describe thermal radiation [29]. These are the following.

\[(i) \ i(\omega, \theta, \phi, T), \ \text{the directional spectral intensity, defined as the energy emitted per unit time, per unit frequency interval d\omega around } \omega, \ \text{per unit elemental projected surface area } dA_{\Omega} \ \text{normal to the direction } \Omega \ \text{defined by } \theta, \ \phi, \ \text{and into a unit elemental solid angle } d\Omega_{s} \ \text{around the direction } \Omega.\]
(ii) \( e(\omega, \theta_s, \phi_s, T) \), the directional spectral emissive power defined as the energy emitted per unit time, per unit frequency interval \( d\omega \) around \( \omega \), per unit elemental surface area \( dA \), and into a unit elemental solid angle \( d\Omega_s \) around the direction \( s \),

\[
e(\omega, \theta_s, \phi_s, T) = i(\omega, \theta_s, \phi_s, T) \cos \theta_s,
\]

(3.6)

(iii) \( I(\omega, T) \), the average spectral intensity, defined as the average energy per unit time, per unit frequency interval \( d\omega \) around \( \omega \), per unit elemental projected surface area \( dA_p \), emitted into the \( 4\pi \) solid angle,

\[
I(\omega, T) = \frac{1}{4\pi} \int_0^{2\pi} d\phi_s \int_0^{\pi/2} \sin \theta_s d\theta_s i(\omega, \theta_s, \phi_s, T).
\]

(3.7)

(iv) \( E(s, \omega, T) \), the hemispherical spectral emissive power, defined as the energy per unit time, per unit frequency interval \( d\omega \) around \( \omega \), per unit elemental surface area \( dA \) normal to the direction \( s \), emitted into the positive \( s \) semi-plane,

\[
E(s, \omega, T) = \int_0^{2\pi} d\phi_s \int_0^{\pi/2} \sin \theta_s d\theta_s e(\omega, \theta_s, \phi_s, T).
\]

(3.8)

(v) \( I(T) \), total intensity, defined as the energy per unit time, per unit elemental projected surface area \( dA_p \), emitted into the positive \( s \) semi-plane,

\[
I(T) = 4\pi \int_{\omega} d\omega I(\omega, T)
\]

\[
= \int_{\omega} d\omega \int_0^{2\pi} d\phi_s \int_0^{\pi/2} \sin \theta_s d\theta_s i(\omega, \theta_s, \phi_s, T).
\]

(3.9)

(vi) \( E(s, T) \), hemispherical total emissive power, defined as the energy per unit time, per unit elemental surface area \( dA \), emitted into the positive \( s \) semi-plane,

\[
E(s, T) = \int_{\omega} d\omega E(s, \omega, T).
\]

(3.10)

In view of the fact that, for a specific Bloch mode, the group velocity and its \( k_n \)-vector index may have different orientations, the evaluation of the angle-dependent spectral intensity and emissive power raises the conceptual problem of how to map the reciprocal space representation of, say, the averaged energy current (3.5a) onto the desired real-space quantities. To facilitate this, we introduce

(i) \( i_{n,k}(T) \), the directional mode intensity defined by the energy per unit time, per mode \( [n,k] \), per unit elemental projected surface area \( dA_p \) normal to the direction \( e_{n,k} \) (defined by \( \theta_{e_n}, \phi_{e_k} \)).

(ii) \( e_{n,k}(T) \), the directional mode emissive power defined by the energy per unit time, per mode \( [n,k] \), per unit elemental surface area \( dA \) normal to the direction \( e_{n,k} \) (defined by \( \theta_{e_k}, \phi_k \)),

\[
e_{n,k}(T) = i_{n,k}(T) \cos \theta_{e_k}.
\]

(3.11)

Then, the total intensity and hemispherical total emissive power can be represented as

\[
I(T) = \sum_{n,k} i_{n,k},
\]

(3.12)

\[
E(s, T) = \sum_{[n,k]} e_{n,k} = \sum_{n,k} i_{n,k} \cos \theta_{e_n,k}.
\]

(3.13)

Here, we have chosen the \( z \)-axis of our system to be oriented along \( s \) and have introduced the restricted summation \( \sum' \) over modes for which \( e_{n,k} \cdot \hat{s} \geq 0 \).

With the help of (3.5b) of \( i_{n,k}(T) \), we thus obtain

\[
I(T) = \sum_{n,k} \hbar \omega_{n,k} v_{n,k}(T) v_{n,k}.
\]

(3.14)

\[
E(s, T) = \sum_{n,k} \hbar \omega_{n,k} v_{n,k}(T) v_{n,k} \cdot \hat{s}.
\]

(3.15)

At this point, we want to emphasize the fact that, in general, the group velocity cannot be represented as a functional of the dispersion relation

\[
v^{(g)}_{n,k} \equiv \nabla \omega(n,k) \neq \nabla \omega(n,k).
\]

This means that it is not possible to convert the expressions of the radiation flux in equations (3.14) and (3.15) into a simple frequency integral involving only the photonic DOS. Rather, in order to arrive at spectral representations we have to employ a procedure similar to the introduction of the local DOS [26] in spontaneous emission calculations: we insert the identity

\[
\int_{\omega} d\omega b(\omega - \omega_{n,k})
\]

into equations (3.14) and (3.15) and convert the \( k \)-space integration via the well known rules for surface \( \int_{\Sigma} = \int_{\Sigma_1} + \int_{\Sigma_2} \) and volume \( \int_{V} = \int_{V_1} + \int_{V_2} \) integrals.

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We now turn to an investigation of the angular resolution of the intensity and the hemispherical emissive power via the evaluation of the associated directional quantities. Basically, we follow the same procedure as in [30] in order to account for the fact that the intensity and the emissive power have a natural angular resolution. The directional mode intensity and mode emissive power defined in equations (3.5b) and (3.11) characterize the flow of thermal radiation in the direction of the mode’s group velocity \( \mathbf{v}_{n,k} \). The key concept in this analysis is the notion of stationary points [30]. For a given position-space direction \( \mathbf{r} \rightarrow \{ \theta_r, \phi_r \} \), we define the set of stationary points \( \{ \mathbf{k}^n_{\nu} \} \) as the set of modes (labeled by band index \( n \) and \( k \)-vectors) that belong to the equi-frequency surface \( \omega = \text{const} \) and whose mode group velocity points in a prescribed direction \( \mathbf{r} \).

For a given real-space direction \( \mathbf{r} \) and a given frequency \( \omega \) it is possible to have zero, one or more stationary points. Clearly, the absence of a stationary point corresponds to a stop band along the corresponding \( k \)-space direction.

We then rewrite the area of the iso-frequency surface, \( A(\Sigma_\omega) \), as a sum over angular integrals around the stationary points (labeled by \( \nu \)) on the iso-frequency surface

\[
A(\Sigma_\omega) = \sum_n \int_{\Sigma_n(\omega)} d\mathbf{k}_n = \sum_n \int \frac{d\Omega_r}{\Omega_1} \frac{1}{k_{\nu}^n}.
\]

A similar analysis for the projected surface area \( A'_{\nu}(\Sigma_\omega) \) yields

\[
A'_{\nu}(\Sigma_\omega) = \sum_n \int \frac{d\Omega_r}{\Omega_1} \frac{\cos \theta_r}{k_{\nu}^n}.
\]

The directional spectral intensity and spectral power can now be expressed in a form that does not depend on the photonic DOS but rather on the geometric details of the photonic band structure

\[
i(\omega, \theta_r, \phi_r, T) = \frac{\hbar \omega(\omega, T)}{\Omega_d(2\pi)^d} \sum_{\nu} \frac{1}{k_{\nu}^n},
\]

\[
e(\omega, \theta_r, \phi_r, T) = \frac{\hbar \omega(\omega, T)}{(2\pi)^d} \cos \theta_r \sum_{\nu} \frac{1}{k_{\nu}^n},
\]

and it is straightforward to verify

\[
I(\omega, T) = \Omega_d \int d\Omega_r i(\omega, \theta_r, \phi_r, T),
\]
Furthermore, in the case of a homogeneous dielectric medium with a linear dispersion relation, the directional spectral intensity and directional spectral emissive power, \(E(\omega, r, T)\) and \(I_d,3(\omega, \theta, \phi, r, T)\), may be readily evaluated, and we have

\[
E(\omega, r, T) = \int_{Q_d/2} d\Omega_e \, e(\omega, \theta_e, \phi_e, T). \tag{3.27b}
\]

A particular situation which deserves special attention is the case of a one-dimensional photonic system which is characterized by only one direction of propagation and can be analyzed within the formalism developed above. For \(d = 1\) \((\Omega_d = 2, \Omega_{p.d} = 1, n_\omega = 1)\), the photonic DOS becomes

\[
\rho_{id}(\omega) = \frac{1}{\pi} \frac{1}{\omega^{3/2}}. \tag{3.29}
\]

We notice that such systems, the spectral energy density and the local spectral energy density depend on the dispersion relation

\[
U_{id}(\omega, T) = U_{id}(\omega, r, T) = \frac{1}{\pi} h\omega (\omega, T) \frac{1}{\omega^{3/2}}. \tag{3.30}
\]

In contrast, the spectral intensity and spectral hemispherical power are independent of the dispersion relation and, therefore, do not depend on the photonic reservoir. Instead, we obtain a universal functional dependence on the frequency \(\omega\)

\[
I(\omega, T) = \frac{1}{2\pi} h\omega (\omega, T), \tag{3.31a}
\]

\[
E(s, \omega, T) = \frac{1}{2\pi} h\omega (\omega, T). \tag{3.31b}
\]

This universality of the radiation flux in a one-dimensional channel has first been pointed out by Pendry [31] in the context of the quantum limits to the flow of information and entropy in a medium comprising several one-dimensional channels. Despite this apparent cancelation of the effects of enhanced photonic DOS and reduced group velocity (which give rise to the universality of equation (3.31)), the presence of a non-trivial dispersion relation in one-dimensional photonic reservoir (such as a photonic crystal) may profoundly influence the dynamics of light–matter interaction. The radiation from elementary emitters/absorbers embedded in a one-dimensional photonic crystal exhibits a very dramatic modification of its temporal evolution and the following physical picture emerges: due to an enhanced photonic DOS, the spontaneous emission rate can be dramatically increased and, once emitted, the radiation propagates very slowly. A very fast decay of an excited atom may be extremely advantageous in minimizing the detrimental effects of other non-radiative decay mechanisms. In a realistic configuration, the atomic system may lose its excitation by transferring it to other degrees of freedom that may be present in the dielectric host (such as phonons). If the photonic crystal-enhanced radiative decay rate becomes much larger than the non-radiative decay rates, non-radiative relaxation is suppressed. On the other hand, a small group velocity for the emitted radiation in the photonic crystal determines a longer interaction time of the radiation with other elementary absorbers/emitters so that enhanced absorption and nonlinear effects are expected.

4. Results

In order to illustrate the potential of the formalism that we have developed in the previous sections, we consider a thermal radiation in a two-dimensional model photonic-crystal system that consists of a square lattice of dielectric cylinders with \(\epsilon = 8.41\) and radius \(r/a = 0.15\), where \(a\) denotes the lattice constant. For radiation whose electric field is polarized parallel to the cylinder axis (E-polarization), this structure exhibits several stop bands and a PBG that, in dimensionless frequencies, extends from \(\omega a/2\pi c = 0.385\) to \(\omega a/2\pi c = 0.48\). As a result, this system allows us to study several features of thermal radiation which are generic to photonic crystals (such as the effects of stop bands and PBGs) and are not limited to this particular model.

In the right panel of figure 1, we display the dispersion relation for our model system. In addition, the left panel shows the dispersion relation for a homogeneous dielectric medium that, in the long-wavelength limit, exhibits the same index of refraction, \(n = 1.53\). In order to facilitate a comparison with the band structure of our model photonic crystal, the linear dispersion relation of the homogeneous system has been folded back into the corresponding 1BZ. For small frequencies, the dispersion relations are essentially the same for both systems. However, as the frequency increases, the photonic-crystal dispersion relation becomes strongly anisotropic, directional stop bands appear and, ultimately, a PBG opens up.

As discussed in section 3, the iso-frequency contours are of paramount importance in understanding the behavior of the thermal radiation flow in such systems. Therefore, we depict in figure 2 the iso-frequency contours for three selected frequencies, \(\omega a/2\pi c = 0.25, 0.3271, 0.345\), and three different physical systems. The left panel features free space, the center panel displays the results for the homogeneous medium whose full dispersion relation has been introduced in figure 1, and the right panel shows the results for our model photonic crystal. Clearly, the results for free space and the homogeneous material differ only by the factor \(n = 1.523\). The case of the photonic crystal is markedly different: while for low frequencies the results are essentially the same as those of the homogeneous medium, we observe that for higher frequencies the iso-frequency contours are topologically disconnected, signaling the existence of directional stop bands. We further notice the existence of points with very low curvature on the iso-frequency contour for higher frequency, which will lead to a angular focusing of the thermal radiation flow as described in section 3. These iso-frequency contours of figure 2 provide the basis for the quantitative analysis of the surface area projected surface areas, \(A(\Sigma_w)\) and \(A'_s(\Sigma_w)\), respectively (see also equations (3.17)). For the corresponding
Figure 1. Dispersion relations for a homogeneous medium with index of refraction $n = 1.523$ (left panel) and a two-dimensional square lattice of dielectric cylinders ($\epsilon = 8.41, r/a = 0.15$) in air (right panel). The index of the homogeneous medium has been chosen such that in the long-wavelength limit both systems exhibit the same linear dispersion.

Figure 2. From left to right: iso-frequency contours for free space, a homogeneous dielectric with dielectric constant equal to $\epsilon = 1.523$ and a square lattice of dielectric cylinders ($\epsilon = 8.41$) in air (with a volume-averaged dielectric constant of $\epsilon_{\text{av}} = 1.523$). Each plot displays three contours for $\omega/\omega_0 = 0.25, 0.3271, 0.345$ (from inside to outside).

Numerical calculations we employ standard $k$-space integration techniques \cite{28} as well as the so-called $kp$-perturbation theory for quantities that involve $k$-derivatives \cite{33, 34}.

For our two-dimensional model system the surface and projected surface areas are, of course, given by the total and projected length of the iso-frequency contour. In figure 3, we depict the results of this evaluation for the three systems alluded to above. It is worth noting that for frequencies near the lower edge of the first stop band the contour length exceeds even the case of a homogeneous medium with the same long-wavelength limit (see the discussions in sections 1 and 2). More precisely, exactly at the point where the first directional stop band appears ($\omega/\omega_0 = 0.3271$), the contour length of the iso-frequency surface is maximized. For still higher frequencies, the iso-frequency contours become topologically disconnected, their length rapidly decreases and vanishes at the photonic band-edge. For frequencies above the PBG, the iso-frequency contours turn into multi-valued surfaces, consisting of a set of open contours associated with different bands. Accordingly, the possibility of assembling a maximal length contour is strongly reduced and the values of the surface area remain below that of a homogeneous dielectric. This suggests that thermal emission in our model photonic crystal is most efficient near the opening of the first stop band and exceeds that of the homogeneous medium—albeit by a small margin. This finding has to be contrasted with the results of a comparison of the corresponding photonic DOS, which we provide in figure 4. On the basis of a DOS-based analysis, one would erroneously conclude that the most efficient thermal emission characteristics of our model photonic crystal would occur near any of the frequencies that are associated with the van Hove singularities inside the bands. Furthermore, even for the first band, the value of the actual thermal emission enhancement as provided by the DOS-based (or equivalently spectral-energy-density-based analysis of figure 4) is much larger than the correct value provided by figure 3. Next, we analyze the angular behavior of the intensity and the hemispherical emissive power by evaluating the corresponding directional...
quantities. For homogeneous dielectrics, the directional spectral intensity and directional spectral emissive power can again be evaluated in closed form and reduce to well known results [29]. In photonic crystals, these quantities are governed by the details of the topology of the dispersion surface. At long wavelengths, the departure of the equi-frequency contour from a homogeneous medium is minimal (see figure 1) and the directional spectral intensity and emissive power (see figures 5 and 6) exhibit a Lambertian dependence. As the frequency increases, the thermal radiation becomes more and more anisotropic. Eventually, the directional stop band appears and the iso-frequency contours within the first BZ become disconnected. As a result, the iso-frequency contours have zero slope normal to the Brillouin zone boundary, and there always exist points of zero curvature along these open equi-frequency contours. This implies that the thermal radiation becomes divergent along certain real-space directions (see figures 5 and 6). This behavior is similar to the focusing of dipole radiation in photonic crystals [30] as well as phonon focusing in ordinary crystals [32] and underscores that the spectral and angular redistributions of photonic states play equally important roles in determining the characteristics of thermal radiation flux in photonic crystals. In particular, this analysis suggests that in realistic systems one would have to worry about local heating effects that arise from the strongly anisotropic energy flux within the photonic crystal. In order to interpret experimental results on thermal emission in photonic crystals, it is, therefore, of paramount importance to reliably determine the spatially resolved temperature profile within the sample.

5. Conclusions

In summary, we have analyzed the radiative properties of a photon gas in a photonic crystal in (global) thermal equilibrium. We have considered the specific configurations
of an ‘infinite’ photonic-crystal sample and have compared this to an ‘infinite’ homogeneous medium with the same long-wavelength properties. We find that the efficiency of the thermal radiation flux may exceed the values of the homogeneous dielectric near the first stop band inside the first band. Higher-order bands do not provide such an enhancement. Furthermore, we have shown that an analysis of thermal radiation that is based on the photonic crystal’s photonic DOS may lead to erroneous results, both in terms of the most efficient frequency ranges and the actual values of the enhancements. Furthermore, we have analyzed the direction dependence of the thermal radiation flux. For specific spectral ranges there are spatial directions in which there are no propagating photonic modes and, consequently, the thermal radiation in these directions is absent. On the other hand, Bloch modes with different wavevectors can have almost the same group velocity. Consequently, the radiation flux associated with these Bloch modes concentrates along specific crystalline directions [30, 32]. Based on this, we have argued that this highly directional radiation flux may present experimental challenges regarding the interpretation of data since local heating effects may compromise analyses based on a global thermal equilibrium.

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